

Exercise 9: TMT4208

Hand out: 22.03.2021

Seminar: 26.03.2021

Hand in: 09.04.2021

Task 1: Stefan-Boltzmann's law – Solar radiation

The total radiated power emitted per m² by a black surface is a function of the surface's absolute temperature:

$$E_b = \sigma T^4 \quad \left[\frac{\text{W}}{\text{m}^2} \right]$$

This equation is known as Stefan-Boltzmann's law (1884) and the constant $\sigma = 5.67 \cdot 10^{-8} \left[\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \right]$ is the Stefan-Boltzmann's constant.

- If the sun may be considered being a black body with an average temperature $T_{\text{sun}} = 5760 \text{ K}$, what is $E_{b,\text{sun}}$?
- The sun's approximate diameter is 1391000 km; what is the total radiated power from the sun?

Task 2: Waste heat in space

For *practical surfaces*, also called *non-ideal surfaces*, the total radiated power emitted per m² is given by:

$$E = \varepsilon E_b = \varepsilon \sigma T^4 \quad \left[\frac{\text{W}}{\text{m}^2} \right]$$

ε is the *emissivity* of the surface and is a non-dimensional parameter in the range 0.00 to 1.00.

Emissivity, ε , of a surface is generally equal to the absorptivity, α , and its relation to the reflectivity, ρ , is given by:

$$\alpha + \rho = 1$$

Imagine you have a system in space that is generating a lot of waste heat which you need to get rid of by radiation into deep, cold space. The power, P_i , you need to dump is $1.0 \times 10^3 \text{ W}$. Your external radiator does not receive incident radiation and has a surface area, A_i , of 1.0 m by 2.0 m. It has useful emission from only one side, and an emissivity of 0.99.

- What is the irradiance, G_i , in this case?
- What is the radiosity, J_i , and how does that relate to the net heat flow $\dot{Q}_{i,\text{net}}$?
- What is the equilibrium temperature of your radiator in °C?

Task 3: Roof temperature

You work part time as a contractor building houses. Your client asks you what difference there is (if any) between using a dark color roof and a light color roof in the south where the summer is very hot (we all know that). The homeowner wants either to use very dark, or very light, shingles with emissivity of 0.90 or 0.10 respectively.

The roof can be considered as a “mathematical surface” with zero thickness, and no heat capacity, etc. Under the roof there is a 0.15 m thick fiberglass insulation and the attic temperature is always maintained at $27^\circ\text{C} \approx 300\text{ K}$. Thermal conduction between the roof and the attic together with radiative energy transfer to or from the surroundings are the main modes of heat transfer to be considered, and the heat flux through the roof is assumed to be described by:

$$|\dot{q}| = k \cdot \Delta T / \Delta s \left[\frac{\text{W}}{\text{m}^2} \right]$$

where $k = 0.040 \left[\frac{\text{W}}{\text{m} \cdot \text{K}} \right]$ and ΔT is the temperature difference between attic and roof

Δs is the insulation thickness

The task here is to determine the equilibrium temperature in the day time for the shingles if the solar power is 1.0 kW/m^2 .

- What is the irradiance, G_i , in the two cases of different emissivity?
- Show that the energy balance pr. square meter roof can be written:

$$\sigma T_{\text{roof}}^4 + \frac{k}{\Delta s \cdot \varepsilon_{\text{roof}}} (T_{\text{roof}} - T_{\text{attic}}) - G_i = 0$$

- How will you determine the roof temperature from the energy balance in b)?
- What are the equilibrium roof temperatures for the two cases of different emissivity?
- What information would you relay to the homeowner about the significance of the equilibrium temperatures?

