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Exercise 8: TMT4208

Hand out: 01.03.2021
Seminar: 05.03.2021
Hand in: 12.03.2021

Task 1: Boundary layer theory - concentration and temperature

Using the same assumptions as in the derivation of the boundary layer equations for momentum (i.e. 2D, stationary, incompressible Newtonian flow with constant properties, constant pressure and characteristic length scales L and δ in x- and y-directions), show that the corresponding governing equations for the thermal- and concentration boundary layers can be written as - provided that you also neglect source terms and the viscous dissipation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{1}$$

$$u\frac{\partial w_A}{\partial x} + v\frac{\partial w_A}{\partial y} = \mathcal{D}\frac{\partial^2 w_A}{\partial y^2} \tag{2}$$

Task 2: Boundary layer theory - heat and mass transfer

As a fluid flows along a plate, various boundary layers will form which shape depend upon the value of relevant dimensionless groups, i.e. Re, Pr and Sc.

a. Starting from Polhausens solution for the temperature field

$$\tilde{T}(\eta) = \frac{T(\eta) - T_s}{T_\infty - T_s} = \frac{I(\eta)}{I(\infty)}, \tag{3}$$

where

$$I(\eta) = \int_0^\eta exp\left(-\frac{1}{2}Pr\int_0^\eta F(\eta)d\eta\right)d\eta \tag{4}$$

and

$$\frac{1}{I(\infty)} \approx 0.332Pr^{1/3}, \qquad \text{for} \qquad 0.6 < Pr < 1000, \tag{5}$$

and

$$\eta = y\sqrt{\frac{u_\infty}{\nu x}} \tag{6}$$

show that the (local) heat flux at the wall surface ($y = \eta = 0$) can be expressed as:

$$-\dot{q}_s = \frac{k\left(T_\infty - T_s\right)}{I(\infty)}\sqrt{\frac{u_\infty}{\nu x}} \tag{7}$$

b. Using the result from sub-task a., together with the definition of convective heat transfer flux;

$$-\dot{q}_s = h'\left(T_\infty - T_s\right), \tag{8}$$

where h' is the *local* heat transfer coefficient, derive Polhausens formula:

$$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}, \tag{9}$$

where

$$Re_x = \frac{u_\infty x}{\nu} \tag{10}$$

is the *local* Reynolds number.

c. Determine the corresponding *average* heat transfer coefficient and corresponding average Nusselt number for a flat plate of length L.

d. How would you proceed to describe *mass transfer* on flat plate described in this task?

Task 3: Heat and mass transfer to air

Air holding 4° flows over a flat plate which is 1.0 m wide and 1.5 m long with a velocity of 1.0 m/s.

a. How much heat must be supplied to the plate in order to maintain it at a uniform temperature of 50°C?

b. How large is is the friction force on the plate?

c. A volatile specie with a diffusion coefficient in air equal to $\mathcal{D} = 2.1 \cdot 10^{-6}$ m^2/s is released from the plate. Is it possible to estimate the mass transfer coefficient in this case? Whould your conclusion change if the diffusion coefficient was $\mathcal{D} = 2.1 \cdot 10^{-4}$ m^2/s?

Note that the material properties of air are not given - this is done deliberately - use relevant sources to find the properties you need!