Department of Material Science and Engineering
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Exercise 7: TMT4208

Hand out: 22.02.2021
Seminar: 26.02.2021
Hand in: 05.02.2021

Task 1: Boundary layer theory

a. Our starting point for the description of the Boundary layer is the Navier-Stokes equations in the following form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

what assumptions have been made to arrive at these?

b. Show that the characteristic length of the boundary layer is

$$\delta \sim \sqrt{\frac{\nu L}{u_\infty}} \tag{3}$$

and that $\delta \ll L$ ultimately requires that $Re_L \gg 1$, where Re_L is the Reynolds number based on the length L. How does this scaling affect the equations described in sub-task a.?

Task 2: Friction and shear stress

Liquid aluminium and slag with a free flow velocity of 0.4 m/s and density of $2.7{\cdot}10^3$ kg/m^3, with dynamic viscosities of $1.0{\cdot}10^{-3}$ and 0.1 Pa s, respectively, are flowing along a plate.

a. Explain what is meant with the (momentum) boundary layer thickness, the (momentum) film thickness and the displacement thickness. How are these so-called boundary layer parameters related to each other?

b. What is the difference between the friction factor and the drag coefficient?

c. Estimate the boundary layer thickness, the friction factor C_f and the wall shear stress τ_w on the plate at 10 cm from the initial sharp edge for each of the materials indicated above.

d. Given that the plate is 0.5x0.5m, estimate the drag coefficient and friction force acting on the plate for each of the materials indicated above.

Task 3: Entrainment of gas in a liquid metal stream

When liquid metal is tapped or poured from one vessel to another, the liquid stream may (in certain cases) be regarded as a solid cylindrical body with radius r_m moving with constant velocity u_m through a stationary gas. Due to the no-slip condition between gas and metal, the moving metal will drag some gas along with it, resulting in a *boundary layer* growing downstream of the tap hole - as sketched in figure 1a. This phenomena, called entrainment, introduces gas into the vessel to which the metal is poured. Evidently, the amount of entrained gas is of importance for further processing and is thus of interest.

The ratio of metal to gas mass *flow* ([kg/s]) is given as

$$\mathcal{R} = \frac{\rho_g u_m \Delta}{\rho_m u_m \pi r_m^2}, \tag{4}$$

where ρ_g and ρ_m represent gas and metal densities, respectively, and Δ represents the cross-sectional area of the *gas* moving alongside the metal stream. Δ can be determined from the graphical representation given in figure 1b (*Glauert & Lighthill, 1954*), but also from estimates derived from the *displacement thickness* of the boundary layer, which for a flat plate is given as:

$$\delta^{**} \approx 1.72\sqrt{\frac{\nu x}{u_m}} \tag{5}$$

a. Describe how δ^{**} should scale compared to r_m if the results for a flat plate are to be applicable for this problem.

b. Show that the cross-sectional area of the displaced gas is

$$\Delta = \pi\left[(r_m + \delta^{**})^2 - r_m^2\right] \tag{6}$$

and calculate $\mathcal{R}$ at $x = L$ based on the above estimate as well as graphically for the two cases given in the table below. Comment on your results based on the arguments you gave in sub-task a.

Table 1: Description of cases for Task 3.

Variable	Symbol	Case 1	Case 2
Stream velocity	u_m	1.0 m/s	2.4 m/s
Gas density	ρ_g	0.34 kg/m^3	0.34 kg/m^3
Metal density	ρ_m	7100 kg/m^3	7100 kg/m^3
Kinematic viscosity of gas	ν	$1.25\cdot10^{-4}$ m^2/s	$1.25\cdot10^{-4}$ m^2/s
Stream length	L	0.8 m	0.4 m
Stream radius	r_m	0.0033 m	0.02 m

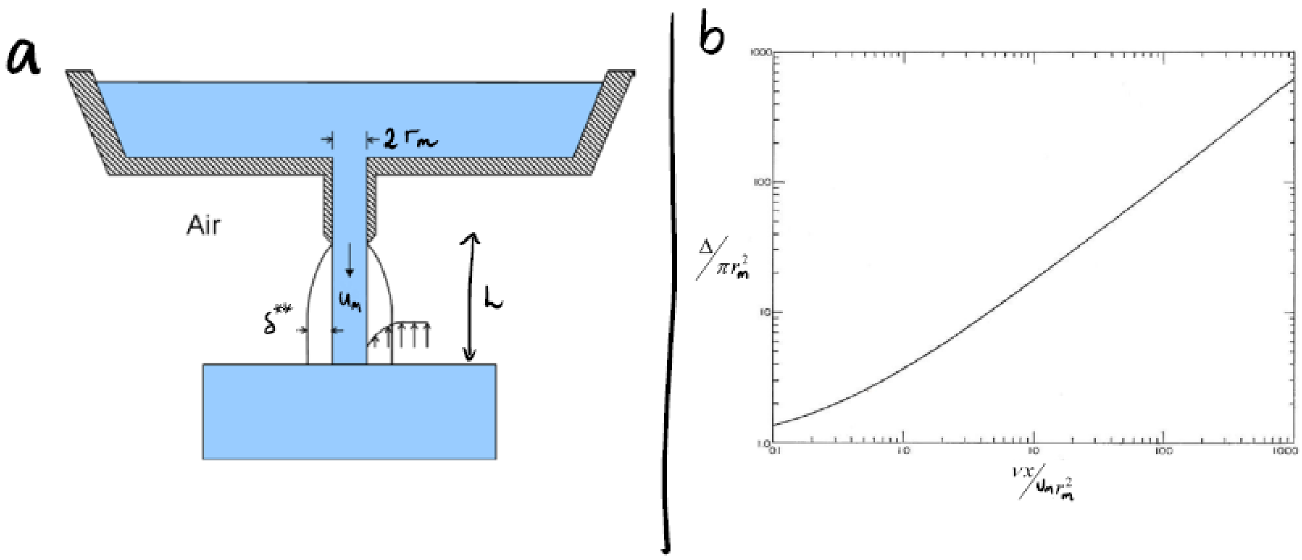

Figure 1: Sketch of tapping process (a) and graphical representation of Δ as of *Glauert & Lighthill, 1954* (b).