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Exercise 6: TMT4208

Hand out: 15.02.2021
Seminar: 19.02.2021
Hand in: 26.02.2021

Task 1: Dimensional and scaling analysis

In the current task, we aim to model the flow of a fluid through a contraction as shown in figure 1, assuming that:

- Steady, 2D flow in the xy-plane
- Newtonian flow with constant ρ and μ
- No body forces
- The average velocity in the upstream section is a known parameter; $\bar{U}_1$
- The (average) outlet pressure is known; $\bar{p}_2$

The (average) inlet pressure $\bar{p}_1$ is unknown and estimating this value is the main goal of this task.

a. Which equations would you start with to solve this problem? Why?

b. Show that the continuity equation and the x-component of Navier-Stokes reduces to:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

c. Introducing the following non-dimensional parameters

$$x^* = \frac{x}{H}, \qquad y^* = \frac{y}{H}, \qquad u^* = \frac{u}{\bar{U}_1} \quad \text{and} \quad v^* = \frac{v}{V_c}, \tag{3}$$

where V_c is a characteristic velocity in the y-direction - show that $V_C = \bar{U}_1$.

d. In order to have a *properly scaled* pressure, the non-dimensional pressure is defined as:

$$p^* = \frac{p - \bar{p}_2}{\Delta p_C}, \tag{4}$$

where Δp_C is a characteristic pressure difference (equal to $\bar{p}_1 - \bar{p}_2$ but more convenient). Describe which requirements must be met by a properly scaled parameter and explain why a scaling on the form

$$p^* = \frac{p}{\bar{p}_2} \tag{5}$$

does not meet these.

e. Show that the Navier-Stokes equation takes the form

$$\left[\frac{\rho \bar{U}_1^2}{H}\right]\left(u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*}\right) = -\left[\frac{\Delta p_C}{H}\right]\frac{\partial p^*}{\partial x^*} + \left[\frac{\mu \bar{U}_1}{H^2}\right]\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right), \tag{6}$$

when introducing the suggested scaled variables.

f. Dividing equation 6 with either $\left[\frac{\rho \bar{U}_1^2}{H}\right]$ or $\left[\frac{\mu \bar{U}_1}{H^2}\right]$ gives a dimensionless equation. Determine the characteristic pressure for each of these cases. Are there any other dimensionless groups present?

g. Assuming that the fluid flowing through the contraction is a liquid metal with the following properties;

$$\mu = 3 \cdot 10^{-3} \text{ Pa s}, \quad \rho = 9 \cdot 10^3 \text{ kg/m}^3, \quad \bar{U}_1 = 0.20 \text{ m/s}, \quad H = 0.010 \text{ m}, \tag{7}$$

argue for how equation 6 should be scaled and determine the characteristic pressure.

h. Assuming that the fluid flowing through the contraction is a molten polymer with the following properties;

$$\mu = 8 \cdot 10^2 \text{ Pa s}, \quad \rho = 1 \cdot 10^3 \text{ kg/m}^3, \quad \bar{U}_1 = 0.20 \text{ m/s}, \quad H = 0.010 \text{ m}, \tag{8}$$

argue for how equation 6 should be scaled and determine the characteristic pressure.

Hint: For sub-tasks g. and h., recall that the equation is *not* properly scaled if any parameter is $\gg 1$, cf. point 6 in the list given in the lecture notes.

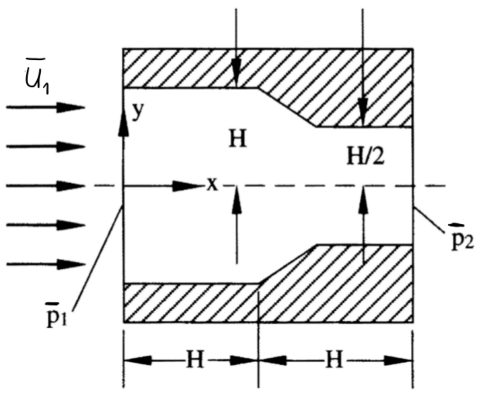

Figure 1: A contracting channel - adapted from *Dantzig & Tucker - Modelling in Materials Processing.*