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Exercise 3: TMT4208

Hand out: 25.01.2021
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Hand in: 05.02.2021

Task 1: Governing equations so far

So far in the course, we have derived equations for conservation of mass, momentum and energy - generally stated as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \tag{1}$$

$$\frac{\partial}{\partial t} (\rho \boldsymbol{u}) + \nabla \cdot (\rho \boldsymbol{u}\boldsymbol{u}) = \rho \boldsymbol{f}_v - \nabla p + \nabla \cdot \boldsymbol{\tau} \tag{2}$$

$$\rho \frac{D\hat{e}}{Dt} = \mu \Phi - \rho p \frac{D\hat{V}}{Dt} - \nabla \cdot \dot{\boldsymbol{q}} + \rho \dot{R} \tag{3}$$

a. Briefly explain the meaning of each of the terms in the above equations.

b. Equation 1 can also be written as:

$$\frac{D\rho}{Dt} + \rho (\nabla \cdot \boldsymbol{u}) = 0 \tag{4}$$

What assumptions must be made in order to arrive at this form?

c. What assumptions are made in order to write the energy equation as:

$$\rho \frac{D\hat{e}}{Dt} = \mu \Phi - \rho p \frac{D\hat{V}}{Dt} + k \nabla^2 T + \rho \dot{R} \tag{5}$$

d. What assumptions are made in order to write the energy equation as:

$$\rho \hat{c}_p \frac{DT}{Dt} = \mu \Phi + k \nabla^2 T + \rho \dot{R} \tag{6}$$

e. Explain why the viscous dissipation function

$$\Phi = \frac{1}{2} \sum_j \sum_i \left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j} \right)^2 \tag{7}$$

always is greater to or equal than zero (limit yourself to 2D cartesian coordinates). What is the consequence of this?

Task 2: The temperature distribution in a falling slag film

In exercise 2, we studied a slag film flowing along a vertical surface and showed that the velocity distribution was given as:

$$u = \frac{gh^2}{\nu}\left(\frac{y}{h} - \frac{1}{2}\frac{y^2}{h^2}\right), \tag{8}$$

where $\nu = 4.1 \cdot 10^{-4}$ m^2/s is the kinematic viscosity of the slag, $h = 5$mm is the film thickness and $g = 9.81$m/s^2 is the acceleration due to gravity, cf. figure 1 in exercise 2.

In this task, we will study the temperature distribution in the slag film, with the following assumptions:

- Steady, 2D flow conditions
- $T = T(y)$
- Incompressible fluid with constant properties
- No heat sources

a. Show that the energy equation under the above assumptions reduces to

$$\frac{\partial^2 T}{\partial y^2} = -\frac{\mu}{k}\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

b. Show that the viscous dissipation function is

$$\Phi = \left(\frac{gh^2}{\nu}\right)\left(1 - 2\frac{y}{h} + \frac{y^2}{h^2}\right) \tag{10}$$

Give a graphical representation of Φ and explain the shape of the function.

c. Show that the general solution for T can be written as

$$T = -\frac{\mu}{k}\left(\frac{gh}{\nu}\right)^2\left(\frac{1}{2}y^2 - \frac{y^3}{3h} + \frac{y^4}{12h^2}\right) + \mathcal{C}_1 y + \mathcal{C}_2 \tag{11}$$

d. The top surface of the slag-film, $y = h$ is assumed to have a constant temperature T_C. Determine the temperature profile when the boundary condition at the bottom surface ($y = 0$) is:

i. $T(y = 0) = T_H$

ii. $\partial T/\partial y|_{y=0} = 0$

What is the physical significance of the last boundary condition?

e. Assuming a density of 3500 kg/m^3 and thermal conductivity of 0.1 W/mK for the slag, $T_C = 300$ K and $T_H = 305$ K, plot the temperature distribution for the two cases and compare to the case without dissipation ($\mu = 0$).