Department of Material Science and Engineering
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Exercise 2: TMT4208

Hand out: 18.01.2021
Seminar: 22.01.2021
Hand in: 29.01.2021

Task 1: Navier Stokes and a falling slag film

A (Newtonian) molten slag with *kinematic* viscosity $\nu = \mu/\rho$ flows down a vertical (stationary and no-slip) wall as sketched in 1. The slag surface is in contact with air at $y = h$ where the boundary condition is

$$\left.\frac{\partial u}{\partial y}\right|_{y=h} = 0 \tag{1}$$

The flow is considered to be incompressible and stationary and the fluid properties and slag film thickness h are considered as being constant. Moreover, the slag layer thickness is assumed to be much smaller that the depth of the wall b (into the paper plane i.e., $b \gg h$) and the flow can this be considered as two-dimensional (i.e. $w \approx 0$ and $u \neq u(z)$).

a. Explain briefly what the boundary condition 1 implies.

b. The Navier-Stokes equation in the x-direction is given as (gravity is acting in the positive x-direction - see figure 1):

$$\frac{\partial}{\partial t}(\rho u) + \nabla \cdot (\rho \boldsymbol{u} u) = \rho g - \frac{\partial p}{\partial x} + \mu \nabla^2 u, \tag{2}$$

and in the y-direction

$$\frac{\partial}{\partial t}(\rho v) + \nabla \cdot (\rho \boldsymbol{u} v) = -\frac{\partial p}{\partial y} + \mu \nabla^2 v, \tag{3}$$

while the continuity equation is given as:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0 \tag{4}$$

Using the information given in this task (and the consequences of this information) it can be shown that the Navier-Stokes equation is reduced to

$$\frac{\partial^2 u}{\partial y^2} = -\frac{g}{\nu}. \tag{5}$$

Show and argue that this is the case.

c. Show that the velocity distribution is expressed as:

$$u = \frac{g}{\nu}\left(hy - \frac{1}{2}y^2\right) \tag{6}$$

d. Determine an expression for the slag flow rate $\dot{V}$.

e. Given $\nu = 4.1 \cdot 10^{-4}$ m^2/s, $b = 0.2$ m and $\dot{V} = 0.72$ m^3/h, determine the thickness of the slag layer.

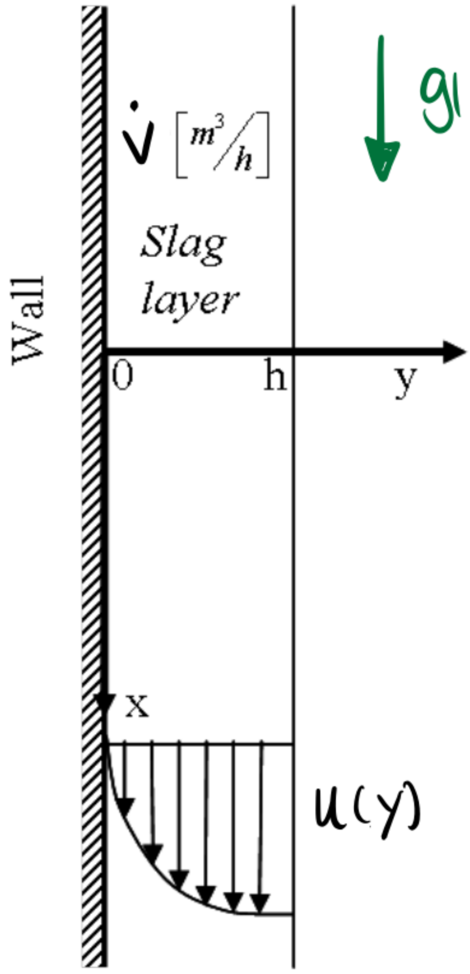

Figure 1: Slag flowing along a vertical wall.