Department of Material Science and Engineering
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Exercise 1: TMT4208

Hand out: 11.01.2021
Seminar: 15.01.2021
Hand in: 22.01.2021

Task 1: Eulerian and Lagrangian frames of reference

a. Let the position of a particle in the Lagrangian frame of reference be denoted as $\boldsymbol{s} = \boldsymbol{s}(\boldsymbol{x}_0, t)$, where $\boldsymbol{x}_0$ is the starting position and t is time. Assuming that the particle is moving with constant velocity u_0 in the $+x$ direction, determine the particle position as a function of time.

b. Assuming that the position as a function of time of material volumes (Lagrangian particles) starting from an initial position x_0 is

$$s(x_0, t) = x_0 (1 + 2t)^{1/2}, \tag{1}$$

determine

i. The velocity of the particles.

ii. The acceleration of the particles.

c. In the Eulerian frame of reference we are aiming to describe the velocity (and other fields) from a fixed perspective, i.e. $\boldsymbol{u}(\boldsymbol{x}, t)$. It can be shown that one knows the Lagrangian velocity, $\boldsymbol{u}_L(\boldsymbol{s}(\boldsymbol{x}_0, t), t)$, that the *Eulerian* velocity can be expressed as $\boldsymbol{u}_E(\boldsymbol{x}, t) = \boldsymbol{u}_L(\boldsymbol{x}(\boldsymbol{s}, t), t)$. We will demonstrate this substitution in a few sub tasks:

i. For the particle described in sub-task b - invert the function to obtain $x_0(s, t)$.

ii. By substitution of the above expression in that of the (Lagrangian) velocity of the particle described in sub-task b and replacing s by x, show that the resulting velocity is:

$$u(x, t) = x (1 + 2t)^{-1} \tag{2}$$

This is the corresponding Eulerian-field, although we will not prove this (directly) here.

d. Determine the acceleration of the fluid in the Eulerian frame of reference.

e. Calculate the total derivative of the Eulerian velocity field and discuss differences (if any) to that found in task b-ii.

Task 2: Mass balances and stream functions

a. For a given a 2D incompressible flow of a fluid with density ρ, the x-component of the velocity is given as

$$u = u_0 + \alpha x, \tag{3}$$

where u_0 and α are constants. Determine the y-component of the velocity, given the boundary condition $v(y = 0) = 0$.

b. Consider a control volume with dimensions as sketched in figure 1a with depth Δz into the paper plane. Calculate the mass flux ($\int_A \rho\,(\boldsymbol{u} \cdot \boldsymbol{n})\,dA$) across each of the (four) surfaces, recalling that the normal vector of a control volume points out of the domain by definition. What is the net mass flux for the control volume?

c. Determine and sketch the stream function for the flow.

d. Would your conclusions in task b have changed if the domain was extended to that shown in figure 1b?

e. Would your conclusions in task b have changed if the velocity field was given as

$$\boldsymbol{u} = \begin{bmatrix} u_0 + \alpha y \\ v_0 + \alpha y \end{bmatrix}? \tag{4}$$

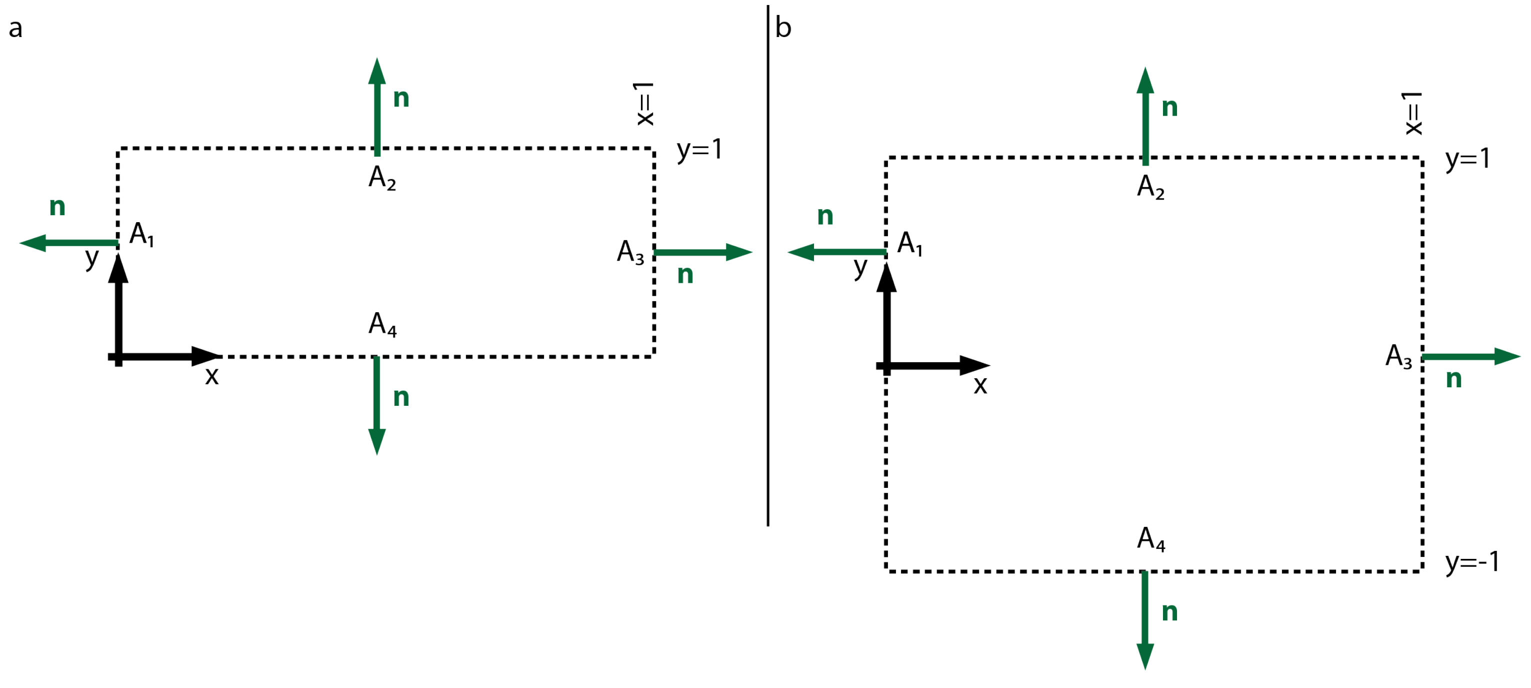

Figure 1: Sketch of control volumes, key coordinates and surfaces for task 2.