Strømning - Øving 1

1a) $s(\vec{x}_0, t) = \vec{x}_0 + \vec{u}_0 t$

1b) $s(x_0, t) = x_0 (1 + 2t)^{1/2}$

i) $u(x_0, t) = \frac{\partial s}{\partial t} = \frac{x_0}{\sqrt{1+2t}}$

ii) $a(x_0, t) = \frac{\partial^2 s}{\partial t^2} = -\frac{x_0}{2(1+2t)^{3/2}}$

1c) i) $x_0 = s(1+2t)^{-\frac{1}{2}}$

ii) $u_E(x, t) = \frac{x(1+2t)^{-\frac{1}{2}}}{\sqrt{1+2t}} = \frac{x}{1+2t}$

1d) $\frac{\partial u_E}{\partial t} = -\frac{2x}{(1+2t)^2} = a_E(x, t)$

1e)

$$\frac{D}{Dt} u_E = -\frac{2x}{(1+2t)^2} + \frac{u_E}{1+2t}$$

$$= -\frac{x}{(1+2t)^2}$$

$$\frac{Du_E}{Dt} = \frac{1}{2} \frac{\partial u_E}{\partial t}$$

Akselerasjonen til et bestemt Fluidelement er $\frac{1}{2}$ av akselerasjonsfeltet.

2a) inkompressibel $\rightarrow \operatorname{div}(\vec{u})=0$

$$\operatorname{div}(\vec{u}) = \alpha + \frac{\partial u_y}{\partial y} = 0$$

$$\frac{\partial u_y}{\partial y} = -\alpha$$

$$u_y = u_{y,0} - \alpha y \;, \quad u_y(y=0) = 0 \rightarrow u_{y,0} = 0$$

$$u_y = -\alpha y$$

2b) $$\Phi_1 = \int_0^1 \rho u(0,y)\hat{n}_1 dy = \underline{\underline{-\rho u_0}}$$

$$\Phi_2 = \int_0^1 \rho u(x,1)\hat{n}_2 dx = \underline{\underline{-\rho\alpha}}$$

$$\Phi_3 = \int_0^1 \rho u(1,y)\hat{n}_3 dy = \rho(u_0+\alpha)$$

$$\Phi_4 = \int_0^1 \rho u(x,0)\hat{n}_4 dx = 0$$

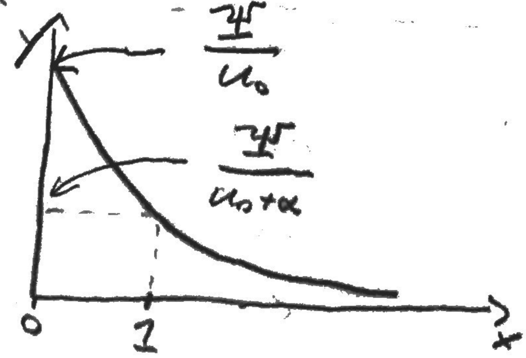
$$\sum \Phi_i = 0 = \int_{cv} \operatorname{div}(u) dV$$

2c) $$\frac{\partial \Psi}{\partial y} \equiv u_x = u_0 + \alpha x \;, \quad \frac{\partial \Psi}{\partial x} \equiv -u_y = \alpha y$$

$$\Psi = u_0 y + \alpha x y + F(x)$$

$$\frac{\partial \Psi}{\partial x} = \alpha y + F'(x) = \alpha y$$

$$\Psi = u_0 y + \alpha x y + C$$



2d) Nei, inkompressibelt Fluid $\rightarrow$ ingen akkumulering

2e) Ja, $\operatorname{div}(u) = \alpha \rightarrow \dfrac{dm_{cv}}{dt} = \displaystyle\int_{cv} \rho\alpha \, dV = 2\rho\alpha$