Strømning, Øving 7

Oppg. 1a)

Stasjonær strømning

(I) $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\partial \rho}{\partial t} = 0$, eller inkompressibel

(II) $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$

$\rightarrow \frac{\partial u}{\partial z} = \frac{\partial v}{\partial y} = 0 \rightarrow$ 2D strømning

$\nabla p = 0$, $f_v = 0$, $\frac{\partial}{\partial t}(\rho u)$

1b) Skalerer med

$x^* = \frac{x}{L}$, $y^* = \frac{y}{\delta}$

$u^* = \frac{u}{u_\infty}$, $v^* = \frac{v}{v_c}$

I) $\frac{u_\infty}{L}\frac{\partial u^*}{\partial x^*} + \frac{v_c}{\delta}\frac{\partial v^*}{\partial y^*}$

$\frac{\partial u^*}{\partial x^*} + \frac{L v_c}{u_\infty \delta}\frac{\partial v^*}{\partial y^*} = 0 \rightarrow v_c = \frac{u_\infty \delta}{L}$

II) $u_\infty u^* \frac{u_\infty}{L}\frac{\partial u^*}{\partial x^*} + \frac{u_\infty \delta}{L} v^* \frac{u_\infty}{\delta}\frac{\partial u^*}{\partial y^*} = \nu\left(\frac{u_\infty}{L^2}\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{u_\infty}{\delta^2}\frac{\partial^2 u^*}{\partial y^{*2}}\right)$

$$\frac{u_\infty^2}{L}\left(u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*}\right) = \frac{\nu u_\infty}{\delta^2}\underbrace{\left(\frac{\delta^2}{L^2}\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}}\right)}_{\approx \frac{\partial^2 u^*}{\partial y^{*2}}}$$

$$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \underbrace{\frac{\nu L}{u_\infty \delta^2}}_{\approx 1}\frac{\partial^2 u^*}{\partial y^{*2}}$$

$\approx 1 \Rightarrow \delta = \sqrt{\frac{\nu L}{u_\infty}}$

$\delta = \sqrt{\frac{\nu L}{u_\infty}} = L\sqrt{\frac{\nu}{u_\infty L}} \rightarrow \frac{\delta}{L} = \sqrt{\frac{1}{Re_L}}$

$\delta \ll L \rightarrow \frac{1}{Re_L} \ll 1 \rightarrow Re_L \gg 1$

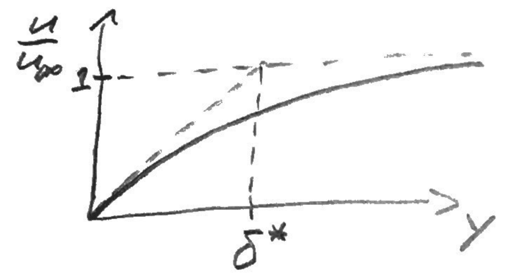
Oppg. 2a)

boundary layer thickness:

$u(y=\delta) = 0{,}99\,u_\infty$

Punktet hvor hastigheten er 99% av free-flow hastighet

Film thickness



tykkelsen til sjiktet dersom hastigheten hadde økt lineært med

$$\frac{du}{dy} = \left.\frac{du}{dy}\right|_{y=0}$$

Displacement thickness

$$\delta^{**} = \int_0^\infty \left(1 - \frac{u}{u_\infty}\right) dy$$

Er relatert til aerodynamikk og trykktap

2b) $$C_D = \frac{1}{\frac{1}{2}\rho u_r^2}\left(\frac{F_D}{A_p^*}\right)$$ ← relatert til motstand når fluid strømmer rundt noe

$$C_F = \frac{1}{\frac{1}{2}\rho u_\infty^2}\tau_w$$ ← relatert til skjærkraft på en flate

2c)

$$\delta = 4{,}92\sqrt{\frac{\nu x}{u_\infty}}$$

$$C_F = 0{,}664\sqrt{\frac{\nu}{u_\infty x}}$$

$$\tau_w = \frac{1}{2}\rho u_\infty^2 C_F$$

i) $\delta = 1{,}5$ mm

ii) $\delta = 1{,}5$ cm

i) $C_F = 2{,}02 \cdot 10^{-3}$

ii) $C_F = 2{,}02 \cdot 10^{-2}$

i) $\tau_w = 0{,}175$ Pa

ii) $\tau_w = 1{,}75$ Pa

i) $\mu = 10^{-3}$ Pas

ii) $\mu = 0{,}1$ Pas

Oppg 2d)

$$F_w = \int \tau_w \, dA = 0{,}5 \int_0^{0,5} \tau_w \, dx$$

$$= 0{,}5 \int_0^{0,5} \frac{1}{2} \rho u_\infty^2 \cdot 0{,}664 \sqrt{\frac{\nu}{u_\infty x}} \, dx$$

$$= \frac{0{,}664}{4} \rho u_\infty^2 \sqrt{\frac{\nu}{u_\infty}} \left[2\sqrt{x} \right]_0^{0,5}$$

$F_w(\mu = 0{,}1\,\mathrm{Pas}) = 0{,}6\,\mathrm{N}$

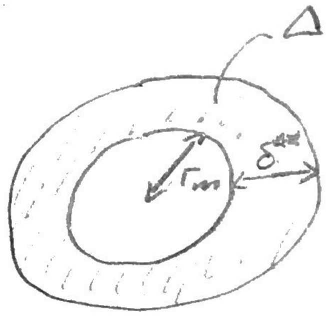
$F_w(\mu = 10^{-3}\,\mathrm{Pas}) = 61{,}7\,\mathrm{mN}$

Oppg. 3a)

$$(\mathrm{I}) \quad R = \frac{\rho_g u_m \Delta}{\rho_m u_m \pi r_m^2} \qquad (\mathrm{II}) \quad \delta^{**} = 1{,}72\sqrt{\frac{\nu x}{u_m}}$$

For at resultater for plate skal være gyldige må $\delta^{**} \ll r_m$

3b)



$$\Delta = \pi(r_m + \delta^{**})^2 - \pi r_m^2 = \pi\left((r_m+\delta^{**})^2 - r_m^2\right)$$

$$\Delta = \pi\left(2 r_m \delta^{**} + (\delta^{**})^2\right) = \pi\delta^{**}(2r_m + \delta^{**}) \approx 2\pi r_m \delta^{**}$$

$$R = \frac{\rho_g u_m 2\pi r_m \delta^{**}}{\rho_m u_m \pi r_m^2}$$

$$R = \frac{2\rho_g \delta^{**}}{\rho_m r_m} = \begin{cases} \text{Case 1}: \underline{5\cdot 10^{-4}} \\ \text{Case 2}: \underline{3{,}76\cdot 10^{-5}} \end{cases}$$

NB: case 1 gir $\delta^{**} = 1{,}72\cdot 10^{-2} > r_m$

$\rightarrow$ Likningen holder ikke

$$\text{Grafisk}: \begin{cases} \text{case 1}: \frac{\nu x}{u_m r_m^2} = 11 \mapsto \frac{\Delta}{\pi r_m^2} \approx 12 \mapsto \underline{R = 5{,}7\cdot 10^{-4}} \\ \text{case 2}: \frac{\nu x}{u_m r_m^2} = 5\cdot 10^{-2} \mapsto \frac{\Delta}{\pi r^2} \approx 1 \mapsto \underline{R = 4{,}7\cdot 10^{-5}} \end{cases}$$