Strømning, Øving 6

1a) Navier-Stokes: Relater trykk, skjærkrefter og hastighetsfelt

Kontinuitetslikning: Relater tetthet til hastighetsfelt

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + (\rho \mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla p + \rho F_v + \nabla \cdot \tau$$

(temp. er ikke umiddelbart interessant her)

1b)

kont: $\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \overset{\text{Steady}}{=} 0$

$$-\rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (I)$$

N.S.: $\underbrace{\rho \frac{\partial u}{\partial t}}_{\substack{=0 \\ \text{Steady}}} + \rho\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + \underbrace{w\frac{\partial}{\partial z}}_{\substack{=0 \\ u = u(x,y)}}\right)u = -\frac{\partial p}{\partial x} + \underbrace{\rho F_{v,x}}_{=0} + (\nabla \cdot \tau)_x$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\nabla^2 u + \frac{\partial}{\partial x}\left(\underbrace{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}}_{\substack{=0 \\ \text{Fra } (I)}} + \underbrace{\frac{\partial w}{\partial z}}_{=0,\ u=u(x,y)}\right)\right)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \underbrace{\frac{\partial^2 w}{\partial z^2}}_{=0,\ u=u(x,y)}\right)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \quad \square$$

1c) $x^* = \frac{x}{H}$; $y^* = \frac{y}{H}$, $u^* = \frac{u}{\bar{U}_1}$, $v^* = \frac{v}{V_c}$

$du = \bar{U}_1 du^*$ $\quad dv = V_c dv^*$

$dx = H dx^*$ $\quad dy = H dy^*$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \frac{\bar{U}_1}{H}\frac{\partial u^*}{\partial x^*} + \frac{V_c}{H}\frac{\partial v^*}{\partial y^*} = 0$$

$$\underbrace{\frac{\partial u^*}{\partial x^*}}_{O(1)} + \frac{V_c}{\bar{U}_1}\underbrace{\frac{\partial v^*}{\partial y^*}}_{O(1)} = 0$$

$$\frac{V_c}{\bar{U}_1} = 1 \rightarrow V_c = \bar{U}_1 \quad \square$$

1d) "Properly Scaled": Må være dimensjonsløs og ligge på intervallet (0, 1)

$$p^* = \frac{p}{\bar{p}_2} \rightarrow p^*(\bar{p}_1) = \frac{\bar{p}_1}{\bar{p}_2} \leftarrow \text{kan ~~være~~} > 1$$

$$p^* = \frac{p - \bar{p}_2}{\Delta p_c} \rightarrow dp = \Delta p_c\, dp^*$$

(Fra 1c: $dv = \bar{U}_1 dv^*$)

$$\rho\left(\bar{U}_1 u^* \frac{\bar{U}_1}{H}\frac{\partial u^*}{\partial x^*} + \bar{U}_1 v^* \frac{\bar{U}_1}{H}\frac{\partial u^*}{\partial x^*}\right) = -\frac{\Delta p_c}{H}\frac{\partial p^*}{\partial x^*} + \mu\left(\frac{\bar{U}_1}{H^2}\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\bar{U}_1}{H^2}\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

$$\frac{\rho \bar{U}_1^2}{H}\left(u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial x^*}\right) = -\frac{\Delta p_c}{H}\frac{\partial p^*}{\partial x^*} + \frac{\mu \bar{U}_1}{H^2}\left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

1F) $u^*\frac{du^*}{dx^*} + v^*\frac{du^*}{dy^*} = -\underbrace{\frac{\Delta P_c}{\rho \bar{U}_1^2}}_{O(1)}\frac{dp^*}{dx^*} + \frac{\nu}{H\bar{U}_1}\left(\frac{d^2u^*}{dx^{*2}} + \frac{d^2v^*}{dy^{*2}}\right)$ (I)

$O(1) \rightarrow \underline{\underline{\Delta P_c = \rho \bar{U}_1^2}}$

(II) $\frac{\bar{U}_1 H}{\nu}\left(u^*\frac{du^*}{dx^*} + v^*\frac{du^*}{dy^*}\right) = -\underbrace{\frac{H\Delta P_c}{\mu \bar{U}_1}}_{O(1)}\frac{dp^*}{dx^*} + \frac{d^2u^*}{dx^{*2}} + \frac{d^2v^*}{dy^{*2}}$

$O(1) \rightarrow \Delta P_c = \frac{\mu \bar{U}_1}{H}$

$Re = \frac{\bar{U}_1 H}{\nu} = \frac{\rho \bar{U}_1 H}{\mu}$ dukker opp i begge tilfeller

1g) $\rho >> \mu \rightarrow \nu << 1$

Tilfelle (I) gir mening, da kanselleres siste ledd.

$\underline{\Delta P_c = \rho \bar{u}_1^2 = 40 \, Pa}$

1h) $\frac{\rho}{\mu} >> 1$, $\bar{U}_1 H << 1 \rightarrow \frac{\bar{U}_1 H}{\nu} << 1$

Velger (II) for at venstre side skal bli neglisjerbar

$\underline{\Delta P_c = \frac{\mu \bar{U}_1}{H} = 16 \, kPa}$