# Strømning øving 4

Oppg. 1a)

$$\dot{n}_A = -cD\nabla x_A + c\tilde{u}x_A$$

$\tilde{u}$ er molar strømningshastighet

$$\tilde{u} = \frac{1}{c}(\dot{n}_A + \dot{n}_B)$$

$$u = \frac{1}{\rho}(\dot{m}_A + \dot{m}_B) = \frac{1}{\rho}(1-r)\dot{m}_A, \quad \dot{m}_B = -r\dot{m}_A$$

$$\tilde{u} = \frac{1}{c}\left(\frac{\dot{m}_A}{M_A} + \frac{\dot{m}_B}{M_B}\right) = \frac{1}{c}\left(1 - \frac{M_A}{M_B}r\right)\dot{m}_A$$

$$\tilde{u} = u \rightarrow \frac{1}{\rho}(1-r) = \frac{1}{c}\left(1 - \frac{M_A}{M_B}r\right)$$

$$-r\left(\frac{1}{\rho} - \frac{1}{c}\frac{M_A}{M_B}\right) = \frac{1}{c} - \frac{1}{\rho}$$

$$\tilde{u} = u \text{ når } r = \frac{c-\rho}{c - \rho\frac{M_A}{M_B}}$$

1b)

$$\dot{n}_A = -cD\nabla x_A + \underbrace{c\frac{1}{c}\dot{n}_A x_A}$$

$\rightarrow n_B = 0$

$$\dot{n}_A(1-x_A) = -cD\nabla x_A$$

$$\dot{n}_A = \frac{-cD}{1-x_A}\frac{\partial x_A}{\partial z} \quad x_A = x_A(z)$$

1c) $\frac{\partial n_A}{\partial t} = -\nabla \dot{n}_A = 0$, ved steady state (med ingen reaksjoner)

$$0 = -\frac{\partial}{\partial z}\left(\frac{-CD}{1-x_A}\frac{\partial x_A}{\partial z}\right)$$

1d) $\frac{\partial}{\partial z}\left(\frac{CD}{1-x_A}\frac{\partial x_A}{\partial z}\right) = 0$

$$\frac{\partial}{\partial z}\left(\frac{1}{1-x_A}\frac{\partial x_A}{\partial z}\right) = 0, \quad CD \neq 0, \quad \frac{\partial}{\partial z}(CD) = 0$$

$$\frac{1}{1-x_A}\frac{\partial x_A}{\partial z} = C_1$$

$$\frac{dx_A}{1-x_A} = C_1 dz$$

$$-\ln(1-x_A) = C_1 z + C_2$$

$$\ln(1-x_A) = C_1 z + C_2$$

$$\ln(1-x_{A1}) = C_1 z_1 + C_2$$

$$\ln(1-x_{A2}) = C_1 z_2 + C_2$$

$$\ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right) = C_1(z_1 - z_2)$$

$$\underbrace{C_1 = \frac{1}{z_1 - z_2}\ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)}$$

$$\ln(1-x_{A1}) - \frac{z_1}{z_2}\ln(1-x_{A2}) = C_2\left(1 - \frac{z_1}{z_2}\right)$$

$$C_2 = \frac{1}{1-\frac{z_1}{z_2}}\ln\left(\frac{1-x_{A1}}{(1-x_{A2})^{z_1/z_2}}\right)$$

$$\ln(1-x_A) = z\ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)^{\frac{1}{z_1-z_2}} + \frac{z_2}{z_2-z_1}\ln\left[\frac{1-x_{A1}}{(1-x_{A2})^{\frac{z_1}{z_2}}}\right]$$

$$\ln(1-x_A) = \ln\left(\frac{1-x_{A1}}{1-x_{A2}}\right)^{\frac{z}{z_1-z_2}} + \ln\left[\frac{(1-x_{A1})^{\frac{z_2}{z_2-z_1}}}{(1-x_{A2})^{\frac{z_1}{z_2-z_1}}}\right]$$

$$1-x_A = \frac{(1-x_{A1})^{\frac{z_2-z}{z_2-z_1}}}{(1-x_{A2})^{\frac{z_1-z}{z_2-z_1}}}$$

ganger med $\frac{1}{1-x_{A1}}$ og rydder opp potenser

$$\frac{1-x_A}{1-x_{A1}} = \left(\frac{1-x_{A2}}{1-x_{A1}}\right)^{\frac{z-z_1}{z_2-z_1}} \qquad \square$$

Oppg. 2a)

$$\dot n_A = -CD_{AB}\nabla x_A + C\tilde u x_A$$

$$\dot n_B = -CD_{BA}\nabla x_B + C\tilde u x_B$$

$$\dot n_A + \dot n_B = C\tilde u$$

$$C\tilde u = -C(D_{AB}\nabla x_A + D_{BA}\nabla x_B) + C\tilde u(x_A + x_B)$$

$$D_{AB}\nabla x_A + D_{BA}\nabla x_B = 0, \quad \nabla x_B = -\nabla x_A$$

$$D_{AB} = D_{BA} \qquad \square$$

2b) For to volum som utveksler ideell gass

$$\boxed{\begin{array}{c|c} n_A^\circ & n_A^\circ \\ n_B^\circ & n_B^\circ \end{array}} \longmapsto \boxed{\begin{array}{c|c} n_A^\circ + \Delta n_A & n_A^\circ - \Delta n_A \\ n_B^\circ + \Delta n_B & n_B^\circ - \Delta n_B \end{array}}, \quad n_T^\circ = n_A^\circ + n_B^\circ$$

$$dS = R\,d\ln p = R\,d\ln(n_A + n_B)$$

$$\Delta S = R\ln\left(\frac{n_T^\circ + \Delta n_A + \Delta n_B}{n_T^\circ}\right) + R\ln\left(\frac{n_T^\circ - \Delta n_A - \Delta n_B}{n_T^\circ}\right) \geq 0$$

$$\ln\left(\frac{(n_T^\circ + \Delta n_A + \Delta n_B)(n_T^\circ - \Delta n_A - \Delta n_B)}{(n_T^\circ)^2}\right) \geq 0$$

$$\frac{(n_T^\circ)^2 - (\Delta n_A + \Delta n_B)^2}{(n_T^\circ)^2} \geq 1$$

Lurer forresten litt på hvordan man kan gjøre denne oppgaven uten termo, men det kommer kanskje i LF? I såfall kan jeg bare sjekke det der :)

$$(\Delta n_A + \Delta n_B)^2 \leq 0 \rightarrow \dot n_A = -\dot n_B$$

$$\Delta n_A = -\Delta n_B \qquad \rightarrow \underline{\tilde u = 0}$$

Oppg. 2c)

$$\dot{n}_A = -CD\nabla x_A$$

$$\dot{n}_A = -CD\frac{\partial x_A}{\partial z}$$

$$\frac{\partial \dot{n}_A}{\partial z} = -CD\frac{\partial^2 x_A}{\partial z^2} = 0 \rightarrow -CDx_A = C_1 z + C_2$$

$$\left.\begin{array}{l} x_A(z=0) = x_{A\alpha} \\ x_A(z=L) = x_{A\beta} \end{array}\right\} \rightarrow \begin{array}{l} C_2 = x_{A\alpha} \\ C_1 = \dfrac{x_{A\beta} - x_{A\alpha}}{L} \end{array}$$

$$\dot{n}_A = -CDC_1$$

$$\dot{n}_A = \frac{CD}{L}(x_{A\alpha} - x_{A\beta})$$

Oppg. 2d)

$$\frac{d}{dt}x_{A\alpha} = -a\dot{n}_A \quad , \quad \text{Fordi: } n_{T\alpha} = n_{T\beta} = \text{konstant}$$

$$\frac{d}{dt}x_{A\beta} = a\dot{n}_A$$

$$x_{A\alpha}(t=0) = 1 , \quad x_{A\beta}(t=0) = 0$$

$$x_{A\alpha} + x_{A\beta} = \frac{n_{A\alpha} + n_{A\beta}}{n_{T\alpha}} = \frac{n_A}{n_{T\alpha}} = 1 ,$$

$$x_{A\beta} = 1 - x_{A\alpha}$$

$$\frac{dx_{A\alpha}}{dt} = -\frac{aCD}{L}(x_{A\alpha} - (1 - x_{A\alpha}))$$

$$\frac{dx_{A\alpha}}{2x_{A\alpha} - 1} = -\frac{aCD}{L}dt$$

$$\frac{1}{2}\ln(2x_{A\alpha} - 1) = -\frac{aCD}{L}t + C_1$$

$$2x_{A\alpha} - 1 = C_2\exp\left(-\frac{2aCD}{L}t\right)$$

$$x_{A\alpha} = C_3\exp\left(-\frac{2aCD}{L}t\right) + \frac{1}{2}$$

$$x_{A\alpha}(0) = 1 \rightarrow C_3 = \frac{1}{2}$$

$$x_{A\alpha} = 0{,}75 \rightarrow t = -\frac{L}{2aCD}\ln(2x_{A\alpha} - 1)$$

$$\underline{\underline{t = 410\,h}} \qquad \left(C = \frac{P}{RT}\right)$$