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Chapman–Enskog solutions to arbitrary order in Sonine polynomials III: Diffusion, thermal diffusion, and thermal conductivity in a binary, rigid-sphere, gas mixture

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ABSTRACT

The Chapman–Enskog solutions of the Boltzmann equation provide a basis for the computation of important transport coefficients for both simple gases and gas mixtures. These coefficients include the viscosity, the thermal conductivity, and the diffusion coefficient. In a preceding paper (I), for simple, rigid-sphere gases (i.e. single-component, monatomic gases) we have shown that the use of higher-order Sonine polynomial expansions enables one to obtain results of arbitrary precision that are free of numerical error and, in a second paper (II), we have extended our initial simple gas work to modeling the viscosity in a binary, rigid-sphere, gas mixture. In this latter paper we reported an extensive set of order 60 results which are believed to constitute the best currently available benchmark viscosity values for binary, rigid-sphere, gas mixtures. It is our purpose in this paper to similarly report the results of our investigation of relatively high-order (order 70), standard, Sonine polynomial expansions for the diffusion- and thermal conductivity-related Chapman–Enskog solutions for binary gas mixtures of rigid-sphere molecules. We note that in this work, as in our previous work, we have retained the full dependence of the solution on the molecular masses, the molecular sizes, the mole fractions, and the intermolecular potential model via the omega integrals. For rigid-sphere gases, all of the relevant omega integrals needed for these solutions are analytically evaluated and, thus, results to any desired precision can be obtained. The values of the transport coefficients obtained using Sonine polynomial expansions for the Chapman–Enskog solutions converge and, therefore, the exact diffusion and thermal conductivity solutions to a given degree of convergence can be determined with certainty by expanding to sufficiently high an order. We have used *Mathematica*® for its versatility in permitting both symbolic and high-precision computations. Our results also establish confidence in the results reported recently by other authors who used direct numerical techniques to solve the relevant Chapman–Enskog equations. While in all of the direct numerical methods more-or-less full calculations need to be carried out with each variation in molecular parameters, our work has utilized explicit, general expressions for the necessary matrix elements that retain the complete parametric dependence of the problem and, thus, only a matrix inversion at the final step is needed as a parameter is varied. This work also indicates how similar results may be obtained for more realistic intermolecular potential models and how other gas-mixture problems may also be addressed with some additional effort.

1. Introduction

The Chapman–Enskog solutions of the Boltzmann equation provide a basis for the computation of important transport coefficients for both simple gases and gas mixtures [1–15]. The use of Sonine polynomial expansions for the Chapman–Enskog solutions was first suggested by Burnett [16] and has become the general method for obtaining the transport coefficients due to the relatively rapid convergence of this series [1–8,16]. While it has been found that relatively, low-order expansions (of order 4) can provide reasonable accuracy in computations of the transport coefficients (to about 1 part in 1000), the adequacy of the low-order expansions for computation of the slip and jump coefficients associated with gas-surface interfaces still needs to be explored. Also of importance is the fact that such low-order expansions do not provide good convergence (in velocity space) for the actual Chapman–Enskog solutions even though the transport coefficients derived from these solutions appear to be reasonable. Thus, it is of some interest to explore Sonine polynomial expansions to higher orders. In a preceding paper [17], we have shown for simple, rigid-sphere gases (i.e. single-component, monatomic gases) that, indeed, the use of higher-order Sonine polynomial expansions enables one to obtain results of arbitrary precision that are free of numerical error. In a second paper, we have extended our initial simple gas work to

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modeling the viscosity in a binary, rigid-sphere, gas mixture [18]. In this latter paper we reported an extensive set of order 60 results which are believed to constitute the best currently available benchmark viscosity values for binary, rigid-sphere, gas mixtures. It is our purpose in this paper to similarly report the results of our investigation of relatively high-order, standard, Sonine polynomial expansions for the diffusion- and thermal conductivity-related Chapman–Enskog solutions for binary gas mixtures of rigid-sphere molecules. In the following sections we describe the basic theory, the theoretical elements specific to diffusion, thermal diffusion, and thermal conductivity, the solution technique in terms of the Sonine polynomials, the bracket integrals, details related to the specific case of rigid-sphere molecules, and our results.

A part of our motivation with respect to this work has been some of the recently reported results on direct numerical solutions of the linearized Boltzmann equations for rigid-sphere, gas mixtures. In particular, results for the transport coefficients and the Chapman–Enskog solutions have been reported by Takata et al. [19]. Our work provides a benchmark for assessing the precision of some of the numerical results reported by these authors and, indeed, we report some such comparisons that we have made. Our work does have an important distinguishing feature in that, for rigid-sphere gas mixtures, we require no numerical integrations and thus, in principle, results of arbitrary precision can be obtained for any given order of the Sonine polynomial expansions. We note that the computational resources available to us at the present time have permitted expansions to order 70 given the manner in which we have implemented this technique, but even here it has been possible to obtain extrapolated results believed to be precise to 14 or more significant digits for each of the normalized gas mixture transport coefficients (depending upon the specific mass ratios, size ratios, and mole fractions considered) and it is certain that further improvements in the implementation of the technique or the availability of better computational resources will allow even higher-order expansions and greater convergence of the results. Further, we note that in this work we have retained the full dependence of the solutions on the molecular masses, the molecular sizes, the mole fractions, and the intermolecular potential model via the omega integrals, and we have obtained explicit (symbolic) expressions for the necessary matrix elements (derived from the bracket integrals) used in evaluating the coefficients in the Sonine polynomial expansions for the coupled Chapman–Enskog equations. These generalized matrix elements, once determined, need not be determined again. For rigid spheres (or for any other potential model of interest that can be represented via the omega integrals), we can then determine in a straightforward manner a set of matrix elements that are specific to the potential model being used and store them. These specific matrix elements require only the input of the appropriately computed omega integrals which, for rigid spheres, are known exactly such that no numerical integrations are needed. In this fashion, our method requires only a matrix inversion at the final step. This is important, as all that is needed for finding both the transport coefficients and the related Chapman–Enskog solutions for arbitrary, binary, rigid-sphere gas mixtures is precomputed in a general form. Thus, we are able to study parametric dependencies and convergence of our results in an economical and systematic way since, once the matrix elements up to the highest order are computed and stored, we can process results to any order up to this highest order without any new computations of matrix elements being required. Further, since our values for the transport coefficients converge with increasing order, since we can use arbitrarily high numerical precision as needed in *Mathematica*® for the final matrix inversion step, and since we can easily compare results for a given order with the results for immediately preceding orders, we can be confident in our results and the degree of convergence obtained.

2. The basic theory

Following the work and notations of Chapman and Cowling [1], we offer below an abbreviated version of the relevant theory. For an arbitrary, rarefied, gas mixture, one begins with the Boltzmann equations describing the molecular distribution functions of the constituent gases:

$$\left(\frac{\partial}{\partial t} + \mathbf{c}_i \cdot \nabla_r + \mathbf{F}_i \cdot \nabla_{c_i}\right) f_i(\mathbf{r}, \mathbf{c}_i, t) = \sum_j \iiint (f_i' f_j' - f_i f_j) g b \, \mathrm{d}b \, \mathrm{d}\varepsilon \, \mathrm{d}\mathbf{c}_j = \sum_j J(f_i f_j), \tag{1}$$

in which the left-hand side (LHS) is known as the streaming term of the equation which contains the differential streaming operator in the brackets, the right-hand side (RHS) is a sum over what are known as the collision integrals in which $J(f_i f_j)$ is called the collision operator, $f_i(\mathbf{r}, \mathbf{c}_i, t)$ is the molecular distribution function of the i-th constituent, g is the magnitude of the pre-collision relative velocity, $\mathbf{g} = \mathbf{c}_j - \mathbf{c}_i$, b is the 'impact parameter' associated with the binary scattering events, ε is an angle corresponding to the azimuthal orientation of the scattering plane, and $\mathbf{c}$ is the molecular velocity. A prime ($'$) indicates a function of a post-collision velocity while the corresponding lack of a prime indicates a pre-collision velocity dependence, e.g. $f_i = f_i(\mathbf{r}, \mathbf{c}_i, t)$ while $f_i' = f_i(\mathbf{r}, \mathbf{c}_i', t)$. In the summation over the different constituents, scattering between like constituents (i.e. when $i = j$) is treated in the same way as scattering between unlike constituents with the various pre- and post-collision velocities retained as separate variables for purposes of integration. In this circumstance, for clarity, it is common practice to drop the i subscript inside the collision integral in order to facilitate the necessary discrimination between the velocities (i.e. $\mathbf{c}_i \to \mathbf{c}$ and $f_i \to f$). Of course it follows from this that, if one is dealing with a simple gas having only one constituent, one obtains from this process the single Boltzmann equation describing the gas in which $j = 1$ and no subscript is necessary on the LHS:

$$\left(\frac{\partial}{\partial t} + \mathbf{c} \cdot \nabla_r + \mathbf{F} \cdot \nabla_c\right) f(\mathbf{r}, \mathbf{c}, t) = \iiint (f' f_1' - f f_1) g b \, \mathrm{d}b \, \mathrm{d}\varepsilon \, \mathrm{d}\mathbf{c}_1. \tag{2}$$

Equivalent expressions for the above equations are often encountered in which $b \, \mathrm{d}b \, \mathrm{d}\varepsilon$ is expressed as $\alpha_{ij}(g, \chi) \, \mathrm{d}\mathbf{e}'$ or $\sigma_{ij}(g, \chi) \, \mathrm{d}\mathbf{\Omega}$ where χ is the scattering angle (the angle between $\mathbf{g}$ and $\mathbf{g}'$) and $\alpha_{ij}(g, \chi) = \sigma_{ij}(g, \chi)$ is known as the differential collision cross-section which describes the probability per unit time per unit volume that two molecules colliding with velocities, $\mathbf{c}_i$ in $\mathrm{d}\mathbf{c}_i$ and $\mathbf{c}_j$ in $\mathrm{d}\mathbf{c}_j$, will have a relative velocity after collision, $\mathbf{g}' = \mathbf{c}_j' - \mathbf{c}_i'$, that lies within the solid angle, $\mathrm{d}\mathbf{e}' = \mathrm{d}\mathbf{\Omega} = \sin(\chi) \, \mathrm{d}\chi \, \mathrm{d}\varepsilon$.

For the specific case of a binary gas mixture, one expresses the distribution functions f_1 and f_2 in the form:

$$f_1 = f_1^{(0)} + f_1^{(1)} + f_1^{(2)} + \cdots, \tag{3}$$

$$f_2 = f_2^{(0)} + f_2^{(1)} + f_2^{(2)} + \cdots, \tag{4}$$

where the lowest-order approximations are chosen to be:

$$f_1^{(0)} = n_1 \left(\frac{m_1}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m_1}{2kT}(\mathbf{c}_1 - \mathbf{c}_0)^2\right), \tag{5}$$

$$f_2^{(0)} = n_2 \left(\frac{m_2}{2\pi kT}\right)^{3/2} \exp\left(-\frac{m_2}{2kT}(\mathbf{c}_2 - \mathbf{c}_0)^2\right), \tag{6}$$

in which m_1 and m_2 are the molecular masses of the constituent gases, k is Boltzmann's constant, and n_1, n_2, $\mathbf{c}_0$ and T are, in general, arbitrary functions of $\mathbf{r}$ and t. Note that, in choosing the lowest-order approximations to be of this form (which correspond to Maxwellian distributions), one has effectively equated the functions n_1 and n_2 to the number densities of the two gases in the mixture, T to the temperature of the mixture, and $\mathbf{c}_0$ to the mass-velocity of the mixture where, $\mathbf{c}_0 = M_1 x_1 \mathbf{c}_1 + M_2 x_2 \mathbf{c}_2$ in which, $M_i = m_i/m_0$, $m_0 = m_1 + m_2$, $x_i \equiv n_i/n$ denote the proportions by number of the constituent gases in the mixture (the mole fractions), and $n = n_1 + n_2$ is the total molecular number density of the binary mixture.

If one now limits further consideration only to the second approximation (up to $f^{(1)}$), that is equivalent to assuming that the distribution functions for each of the constituents can be expressed as small linear perturbations from equilibrium states specified by the Maxwellian distributions of Eqs. (5) and (6), i.e. $f_i = f_i^{(0)}(1 + \Phi_i^{(1)})$. Thus, $f_1^{(1)}$ and $f_2^{(1)}$ are written in the form:

$$f_1^{(1)} = f_1^{(0)} \Phi_1^{(1)}, \tag{7}$$

$$f_2^{(1)} = f_2^{(0)} \Phi_2^{(1)}, \tag{8}$$

where the perturbations, $\Phi_1^{(1)}$ and $\Phi_2^{(1)}$, satisfy the time derivative expressions given by:

$$\mathcal{D}_1^{(1)} = -n_1^2 I_1\big(\Phi_1^{(1)}\big) - n_1 n_2 I_{12}\big(\Phi_1^{(1)} + \Phi_2^{(1)}\big), \tag{9}$$

$$\mathcal{D}_2^{(1)} = -n_2^2 I_2\big(\Phi_2^{(1)}\big) - n_1 n_2 I_{21}\big(\Phi_1^{(1)} + \Phi_2^{(1)}\big), \tag{10}$$

in which:

$$\mathcal{D}_i^{(r)} = \frac{\partial_{r-1} f_i^{(0)}}{\partial t} + \frac{\partial_{r-2} f_i^{(1)}}{\partial t} + \cdots + \frac{\partial_0 f_i^{(r-1)}}{\partial t} + \left(\mathbf{c}_i \cdot \frac{\partial}{\partial \mathbf{r}} + \mathbf{F}_i \cdot \frac{\partial}{\partial \mathbf{c}_i}\right) f_i^{(r-1)}, \tag{11}$$

and where:

$$n_i^2 I_i(F) = \iint f_i^{(0)} f^{(0)} (F_i + F - F_i' - F') g \alpha_i \, d\mathbf{e}' \, d\mathbf{c}, \tag{12}$$

$$n_i n_j I_{ij}(K) = \iint f_i^{(0)} f_j^{(0)} (K - K') g \alpha_{ij} \, d\mathbf{e}' d\mathbf{c}_j, \tag{13}$$

are linear functions of their arguments such that $I(\phi + \psi) = I(\phi) + I(\psi)$ and $I(a\phi) = aI(\phi)$ regardless of subscripts (where a is any arbitrary constant). The LHSs of Eqs. (9) and (10) can be expressed as:

$$\mathcal{D}_1^{(1)} = f_1^{(0)} \left\{ \left(\mathscr{C}_1^2 - \frac{5}{2} \right) \mathbf{C}_1 \cdot \nabla \ln(T) + x_1^{-1} \mathbf{d}_{12} \cdot \mathbf{C}_1 + 2 \overset{\circ}{\mathscr{C}_1 \mathscr{C}_1} : \nabla \mathbf{c}_0 \right\}, \tag{14}$$

$$\mathcal{D}_2^{(1)} = f_2^{(0)} \left\{ \left(\mathscr{C}_2^2 - \frac{5}{2} \right) \mathbf{C}_2 \cdot \nabla \ln(T) + x_2^{-1} \mathbf{d}_{21} \cdot \mathbf{C}_2 + 2 \overset{\circ}{\mathscr{C}_2 \mathscr{C}_2} : \nabla \mathbf{c}_0 \right\}, \tag{15}$$

in which $\mathscr{C}_i \equiv (m_i/2kT)^{1/2} \mathbf{C}_i$, $\mathbf{C}_i \equiv \mathbf{c}_i - \mathbf{c}_0$, and the bold sans serif notation, w, denotes a dyadic tensor, $\mathsf{w} = \mathbf{ab}$, constructed from the components of the vectors, $\mathbf{a}$ and $\mathbf{b}$. Note that $\mathbf{d}_{12}$ is given as either of the two forms noted below:

$$\mathbf{d}_{12} = \frac{\rho_1 \rho_2}{\rho p} \left\{ \mathbf{F}_2 - \frac{1}{\rho_2} \nabla p_2 - \left(\mathbf{F}_1 - \frac{1}{\rho_1} \nabla p_1 \right) \right\}, \tag{16}$$

$$\mathbf{d}_{12} = \nabla x_1 + \frac{n_1 n_2 (m_2 - m_1)}{n \rho} \nabla \ln(p) - \frac{\rho_1 \rho_2}{\rho p} (\mathbf{F}_1 - \mathbf{F}_2), \tag{17}$$

where $\rho_i = n_i m_i$ are the mass densities of the constituent gases, $\rho = \rho_1 + \rho_2$ is the total mass density of the mixture, p_1 and p_2 are the partial pressures of the constituent gases, and $p = p_1 + p_2$ is the total pressure of the mixture. Since $\nabla x_2 = -\nabla x_1$, either Eq. (16) or Eq. (17) can be used to show that $\mathbf{d}_{21} = -\mathbf{d}_{12}$. The functions $\Phi_1^{(1)}$ and $\Phi_2^{(1)}$ can then be expressed as:

$$\Phi_1^{(1)} = -\mathbf{A}_1 \cdot \frac{\partial \ln(T)}{\partial \mathbf{r}} - \mathbf{D}_1 \cdot \mathbf{d}_{12} - 2\mathsf{B}_1 : \frac{\partial}{\partial \mathbf{r}} \mathbf{c}_0, \tag{18}$$

$$\Phi_2^{(1)} = -\mathbf{A}_2 \cdot \frac{\partial \ln(T)}{\partial \mathbf{r}} - \mathbf{D}_2 \cdot \mathbf{d}_{12} - 2\mathsf{B}_2 : \frac{\partial}{\partial \mathbf{r}} \mathbf{c}_0, \tag{19}$$

where the functions $\mathbf{A}$ and $\mathbf{D}$ are vectors and the functions B are non-divergent tensors, such that:

$$\mathbf{A} = \mathbf{C} A(C), \quad \mathbf{D} = \mathbf{C} D(C), \quad \mathsf{B} = \overset{\circ}{\mathbf{C}\mathbf{C}} B(C), \tag{20}$$

with the appropriate subscript 1 or 2 implied throughout each expression, and where $\mathbf{A}$, $\mathbf{D}$, and B, must satisfy the following pairs of equations, respectively:

$$f_1^{(0)} \left(\mathscr{C}_1^2 - \frac{5}{2} \right) \mathbf{C}_1 = n_1^2 I_1(\mathbf{A}_1) + n_1 n_2 I_{12}(\mathbf{A}_1 + \mathbf{A}_2), \tag{21}$$

$$f_2^{(0)} \left(\mathscr{C}_2^2 - \frac{5}{2} \right) \mathbf{C}_2 = n_2^2 I_2(\mathbf{A}_2) + n_1 n_2 I_{21}(\mathbf{A}_1 + \mathbf{A}_2), \tag{22}$$

$$x_1^{-1} f_1^{(0)} \mathbf{C}_1 = n_1^2 I_1(\mathbf{D}_1) + n_1 n_2 I_{12}(\mathbf{D}_1 + \mathbf{D}_2), \tag{23}$$

$$-x_2^{-1} f_2^{(0)} \mathbf{C}_2 = n_2^2 I_2(\mathbf{D}_2) + n_1 n_2 I_{21}(\mathbf{D}_1 + \mathbf{D}_2), \tag{24}$$

$$f_1^{(0)} \overset{\circ}{\mathscr{C}_1 \mathscr{C}_1} = n_1^2 I_1(\mathsf{B}_1) + n_1 n_2 I_{12}(\mathsf{B}_1 + \mathsf{B}_2), \tag{25}$$

$$f_2^{(0)} \overset{\circ}{\mathscr{C}_2 \mathscr{C}_2} = n_2^2 I_2(\mathsf{B}_2) + n_1 n_2 I_{21}(\mathsf{B}_1 + \mathsf{B}_2). \tag{26}$$

Note that the form of the distribution functions $\Phi_1^{(1)}$ and $\Phi_2^{(1)}$ has been chosen such that $\mathbf{A}$ and $\mathbf{D}$ must also satisfy the relationships:

$$\int f_1^{(0)} m_1 \mathbf{C}_1 \cdot \mathbf{A}_1 \, d\mathbf{c}_1 + \int f_2^{(0)} m_2 \mathbf{C}_2 \cdot \mathbf{A}_2 \, d\mathbf{c}_2 = 0, \tag{27}$$

$$\int f_1^{(0)} m_1 \mathbf{C}_1 \cdot \mathbf{D}_1 \, d\mathbf{c}_1 + \int f_2^{(0)} m_2 \mathbf{C}_2 \cdot \mathbf{D}_2 \, d\mathbf{c}_2 = 0, \tag{28}$$

and the second-order, Chapman–Enskog approximations yield:

$$f_1^{(1)} = f_1^{(0)} \big\{ 1 - A_1(C_1) \mathbf{C}_1 \cdot \nabla \ln(T) - D_1(C_1) \mathbf{C}_1 \cdot \mathbf{d}_{12} - 2B_1(C_1) \overset{\circ}{\mathbf{C}_1 \mathbf{C}_1} : \nabla \mathbf{c}_0 \big\}, \tag{29}$$

and:

$$f_2^{(1)} = f_2^{(0)} \big\{ 1 - A_2(C_2) \mathbf{C}_2 \cdot \nabla \ln(T) - D_2(C_2) \mathbf{C}_2 \cdot \mathbf{d}_{12} - 2B_2(C_2) \overset{\circ}{\mathbf{C}_2 \mathbf{C}_2} : \nabla \mathbf{c}_0 \big\}. \tag{30}$$

Eqs. (29) and (30) then allow one to verify that the mean kinetic energies of the peculiar motions of the molecules of each constituent gas are the same up to this level of approximation.

From Eqs. (21)–(26), one may then construct the following general expressions:

$$n^2 \{\mathbf{A}, \mathbf{a}\} = \int f_1^{(0)} \left(\mathscr{C}_1^2 - \frac{5}{2} \right) \mathbf{C}_1 \cdot \mathbf{a}_1 \, d\mathbf{c}_1 + \int f_2^{(0)} \left(\mathscr{C}_2^2 - \frac{5}{2} \right) \mathbf{C}_2 \cdot \mathbf{a}_2 \, d\mathbf{c}_2, \tag{31}$$

$$n^2 \{\mathbf{D}, \mathbf{a}\} = x_1^{-1} \int f_1^{(0)} \mathbf{C}_1 \cdot \mathbf{a}_1 \, d\mathbf{c}_1 - x_2^{-1} \int f_2^{(0)} \mathbf{C}_2 \cdot \mathbf{a}_2 \, d\mathbf{c}_2, \tag{32}$$

$$n^2 \{\mathsf{B}, \mathsf{b}\} = \int f_1^{(0)} \overset{\circ}{\mathscr{C}_1 \mathscr{C}_1} : \mathsf{b}_1 \, d\mathbf{c}_1 + \int f_2^{(0)} \overset{\circ}{\mathscr{C}_2 \mathscr{C}_2} : \mathsf{b}_2 \, d\mathbf{c}_2, \tag{33}$$

where $\mathbf{a}$ is any vector-function defined in both velocity domains, $\mathbf{b}$ is any tensor function defined in both velocity domains, and $\{F, G\}$ are known as the bracket integrals which are defined as:

$$n^2\{F, G\} \equiv n_1^2[F, G]_1 + n_1 n_2[F_1 + F_2, G_1 + G_2]_{12} + n_2^2[F, G]_2, \quad (34)$$

where:

$$[F, G]_1 \equiv \int G_1 I_1(F)\,\mathrm{d}\mathbf{c}_1, \quad (35)$$

$$[F, G]_2 \equiv \int G_2 I_2(F)\,\mathrm{d}\mathbf{c}_2, \quad (36)$$

and:

$$[F_1 + G_2, H_1 + K_2]_{12} \equiv \int F_1 I_{12}(H_1 + K_2)\,\mathrm{d}\mathbf{c}_1 + \int G_2 I_{21}(H_1 + K_2)\,\mathrm{d}\mathbf{c}_2. \quad (37)$$

Here, due to symmetry and linearity, one has that $[F, G]_1 = [G, F]_1$, $[F, G]_2 = [G, F]_2$, and $[F_1 + G_2, H_1 + K_2]_{12} = [H_1 + K_2, F_1 + G_2]_{12}$ such that $\{F, G\} = \{G, F\}$, $\{F, G + H\} = \{F, G\} + \{F, H\}$, and $\{F, aG\} = a\{F, G\}$ (where a is any arbitrary constant).

3. The theory for diffusion and thermal diffusion

In a gas mixture, diffusion occurs when the constituents of the mixture have different mean velocities such that $\bar{\mathbf{c}}_1 - \bar{\mathbf{c}}_2 = \bar{\mathbf{C}}_1 - \bar{\mathbf{C}}_2 \neq 0$. This implies that:

$$\bar{\mathbf{C}}_1 - \bar{\mathbf{C}}_2 = n_1^{-1}\int f_1 \mathbf{C}_1\,\mathrm{d}\mathbf{c}_1 - n_2^{-1}\int f_2 \mathbf{C}_2\,\mathrm{d}\mathbf{c}_2 \neq 0. \quad (38)$$

Now, in terms of the second approximation distribution functions of Eqs. (29) and (30):

$$\begin{aligned}\bar{\mathbf{C}}_1 - \bar{\mathbf{C}}_2 &= -\frac{1}{3}\bigg[\bigg\{n_1^{-1}\int f_1^{(0)} C_1^2 D_1(C_1)\,\mathrm{d}\mathbf{c}_1 - n_2^{-1}\int f_2^{(0)} C_2^2 D_2(C_2)\,\mathrm{d}\mathbf{c}_2\bigg\}\mathbf{d}_{12} \\ &\quad + \bigg\{n_1^{-1}\int f_1^{(0)} C_1^2 A_1(C_1)\,\mathrm{d}\mathbf{c}_1 - n_2^{-1}\int f_2^{(0)} C_2^2 A_2(C_2)\,\mathrm{d}\mathbf{c}_2\bigg\}\nabla\ln(T)\bigg] \\ &= -\frac{1}{3}\bigg[\bigg\{n_1^{-1}\int f_1^{(0)} \mathbf{C}_1\cdot\mathbf{D}_1\,\mathrm{d}\mathbf{c}_1 - n_2^{-1}\int f_2^{(0)} \mathbf{C}_2\cdot\mathbf{D}_2\,\mathrm{d}\mathbf{c}_2\bigg\}\mathbf{d}_{12} \\ &\quad + \bigg\{n_1^{-1}\int f_1^{(0)} \mathbf{C}_1\cdot\mathbf{A}_1\,\mathrm{d}\mathbf{c}_1 - n_2^{-1}\int f_2^{(0)} \mathbf{C}_2\cdot\mathbf{A}_2\,\mathrm{d}\mathbf{c}_2\bigg\}\nabla\ln(T)\bigg] \\ &= \frac{1}{3}n\big[\{\mathbf{D},\mathbf{D}\}\mathbf{d}_{12} + \{\mathbf{D},\mathbf{A}\}\nabla\ln(T)\big]. \quad (39)\end{aligned}$$

The definition of the diffusion coefficient is obtained from Eq. (39) based on the case when the gas mixture is uniform in temperature and pressure, and when no external forces are acting on the molecules. Under these conditions, $\mathbf{d}_{12} = \nabla x_1 = n^{-1}\nabla n_1$ and $\nabla\ln(T) = 0$ such that:

$$\bar{\mathbf{c}}_1 - \bar{\mathbf{c}}_2 = \bar{\mathbf{C}}_1 - \bar{\mathbf{C}}_2 = \frac{1}{3}\{\mathbf{D},\mathbf{D}\}\nabla n_1. \quad (40)$$

This may then be expressed as:

$$\begin{aligned}\bar{\mathbf{c}}_1 - \bar{\mathbf{c}}_2 &= -D_{12}\big(n_1^{-1}\nabla n_1 - n_2^{-1}\nabla n_2\big) \\ &= -D_{12}\frac{n}{n_1 n_2}\nabla n_1, \quad (41)\end{aligned}$$

in which the constant of proportionality, D_{12}, is the diffusion coefficient. In this manner, it follows that the diffusion coefficient must be defined as:

$$D_{12} \equiv \frac{n_1 n_2}{3n}\{\mathbf{D},\mathbf{D}\}. \quad (42)$$

If there is a temperature gradient present, then the second term in Eq. (39) also contributes in a similar manner allowing the definition of a thermal diffusion coefficient:

$$D_T \equiv \frac{n_1 n_2}{3n}\{\mathbf{D},\mathbf{A}\}, \quad (43)$$

and one then has:

$$\mathrm{k}_T \equiv D_T/D_{12} = \{\mathbf{D},\mathbf{A}\}/\{\mathbf{D},\mathbf{D}\}, \quad (44)$$

which is known as the thermal-diffusion ratio and which allows Eq. (39) to be expressed as:

$$\begin{aligned}\bar{\mathbf{C}}_1 - \bar{\mathbf{C}}_2 &= -\frac{n^2}{n_1 n_2}\big\{D_{12}\mathbf{d}_{12} + D_T\nabla\ln(T)\big\} \\ &= -\frac{n^2}{n_1 n_2}D_{12}\big\{\mathbf{d}_{12} + \mathrm{k}_T\nabla\ln(T)\big\}. \quad (45)\end{aligned}$$

4. The theory for thermal conductivity

Assuming that the molecules in the mixture have only kinetic energy of translation associated with them, i.e. that there are no internal degrees of freedom which can participate in exchanges of energy during collisions, the thermal flux may be expressed as:

$$\mathbf{q} = \int f_1\frac{1}{2}m_1 C_1^2\mathbf{C}_1\,\mathrm{d}\mathbf{c}_1 + \int f_2\frac{1}{2}m_2 C_2^2\mathbf{C}_2\,\mathrm{d}\mathbf{c}_2, \quad (46)$$

such that:

$$\begin{aligned}&\frac{\mathbf{q}}{kT} - \frac{5}{2}(n_1\bar{\mathbf{C}}_1 + n_2\bar{\mathbf{C}}_2) \\ &= \int f_1\left(\mathscr{C}_1^2 - \frac{5}{2}\right)\mathbf{C}_1\,\mathrm{d}\mathbf{c}_1 + \int f_2\left(\mathscr{C}_2^2 - \frac{5}{2}\right)\mathbf{C}_2\,\mathrm{d}\mathbf{c}_2 \\ &= -\frac{1}{3}\int f_1^{(0)}\left(\mathscr{C}_1^2 - \frac{5}{2}\right)\big\{(\mathbf{C}_1\cdot\mathbf{D}_1)\mathbf{d}_{12} + (\mathbf{C}_1\cdot\mathbf{A}_1)\nabla\ln(T)\big\}\,\mathrm{d}\mathbf{c}_1 \\ &\quad - \frac{1}{3}\int f_2^{(0)}\left(\mathscr{C}_2^2 - \frac{5}{2}\right)\big\{(\mathbf{C}_2\cdot\mathbf{D}_2)\mathbf{d}_{12} + (\mathbf{C}_2\cdot\mathbf{A}_2)\nabla\ln(T)\big\}\,\mathrm{d}\mathbf{c}_2 \\ &= -\frac{1}{3}n^2\big[\{\mathbf{A},\mathbf{D}\}\,\mathbf{d}_{12} + \{\mathbf{A},\mathbf{A}\}\nabla\ln(T)\big]. \quad (47)\end{aligned}$$

Rearranging Eq. (47), one has:

$$\mathbf{q} = \frac{5}{2}kT\big(n_1\bar{\mathbf{C}}_1 + n_2\bar{\mathbf{C}}_2\big) - \frac{1}{3}kn^2T\big[\{\mathbf{A},\mathbf{D}\}\mathbf{d}_{12} + \{\mathbf{A},\mathbf{A}\}\nabla\ln(T)\big], \quad (48)$$

which can be expressed as:

$$\begin{aligned}\mathbf{q} &= \frac{5}{2}kT\big(n_1\bar{\mathbf{C}}_1 + n_2\bar{\mathbf{C}}_2\big) + knT\big(\bar{\mathbf{C}}_1 - \bar{\mathbf{C}}_2\big)\big(\{\mathbf{A},\mathbf{D}\}/\{\mathbf{D},\mathbf{D}\}\big) - \lambda\nabla T \\ &= -\lambda\nabla T + \frac{5}{2}kT\big(n_1\bar{\mathbf{C}}_1 + n_2\bar{\mathbf{C}}_2\big) + knT\mathrm{k}_T\big(\bar{\mathbf{C}}_1 - \bar{\mathbf{C}}_2\big), \quad (49)\end{aligned}$$

in which:

$$\begin{aligned}\lambda &\equiv \frac{1}{3}kn^2\big[\{\mathbf{A},\mathbf{A}\} - \{\mathbf{A},\mathbf{D}\}^2/\{\mathbf{D},\mathbf{D}\}\big] \\ &= \frac{1}{3}kn^2\big\{\tilde{\mathbf{A}},\tilde{\mathbf{A}}\big\}, \quad (50)\end{aligned}$$

where:

$$\{\tilde{\mathbf{A}},\tilde{\mathbf{A}}\} = \{\mathbf{A},\mathbf{A}\} - \{\mathbf{A},\mathbf{D}\}^2/\{\mathbf{D},\mathbf{D}\} = \{\mathbf{A},\mathbf{A}\} - 2k_T\{\mathbf{A},\mathbf{D}\} + k_T^2\{\mathbf{D},\mathbf{D}\}, \tag{51}$$

such that:

$$\tilde{\mathbf{A}}_1 \equiv \mathbf{A}_1 - (\{\mathbf{A},\mathbf{D}\}/\{\mathbf{D},\mathbf{D}\})\mathbf{D}_1 = \mathbf{A}_1 - k_T\mathbf{D}_1, \tag{52}$$

and:

$$\tilde{\mathbf{A}}_2 \equiv \mathbf{A}_2 - (\{\mathbf{A},\mathbf{D}\}/\{\mathbf{D},\mathbf{D}\})\mathbf{D}_2 = \mathbf{A}_2 - k_T\mathbf{D}_2. \tag{53}$$

Now, assuming the absence of any mutual diffusion such that $\bar{\mathbf{C}}_1 = \bar{\mathbf{C}}_2 = 0$, $\mathbf{q} = -\lambda\nabla T$ and λ (which is sometimes used to represent the molecular mean free path) is clearly identical to the thermal conductivity coefficient (which is sometimes also denoted by they symbol κ). The first term in the RHS of Eq. (49) is the heat flow due to inequalities of temperature in the mixture, i.e. the steady-state heat flow when temperature gradients are maintained. The second term in the RHS of Eq. (49) only occurs because the thermal energy and flow are measured relative to $\mathbf{c}_0$ and represents heat flow carried along by the molecular flux in the presence of diffusion. The third term in the RHS of Eq. (49) represents the diffusion thermo-effect which is the analogous inverse process to thermal diffusion.

5. Solution in terms of Sonine polynomials

For the Chapman–Enskog coefficients of diffusion, thermal diffusion, and thermal conductivity, it is necessary to evaluate the bracket integrals, $\{\tilde{\mathbf{A}},\tilde{\mathbf{A}}\}$, $\{\mathbf{A},\mathbf{D}\}$, and $\{\mathbf{D},\mathbf{D}\}$. It is assumed that the Chapman–Enskog functions, $\tilde{\mathbf{A}}_1$, $\tilde{\mathbf{A}}_2$, $\mathbf{D}_1$, and $\mathbf{D}_2$, may be expanded as:

$$\tilde{\mathbf{A}}_1 = \sum_{\substack{p=-\infty\\p\neq 0}}^{+\infty} a_p\mathbf{a}_1^{(p)}, \qquad \tilde{\mathbf{A}}_2 = \sum_{\substack{p=-\infty\\p\neq 0}}^{+\infty} a_p\mathbf{a}_2^{(p)}, \tag{54}$$

and:

$$\mathbf{D}_1 = \sum_{p=-\infty}^{+\infty} d_p\mathbf{a}_1^{(p)}, \qquad \mathbf{D}_2 = \sum_{p=-\infty}^{+\infty} d_p\mathbf{a}_2^{(p)}, \tag{55}$$

where, following the notations of Chapman and Cowling [1] (Note: In Chapman and Cowling, summations that explicitly omit the 0-th term are denoted with primes on the summation symbol), one has:

$$\begin{aligned}
&\mathbf{a}_1^{(p)} = \mathbf{a}_1^{(0)} \equiv M_1^{1/2}\rho_2\boldsymbol{\mathscr{C}}_1/\rho && (p=0),\\
&\mathbf{a}_2^{(-p)} = \mathbf{a}_2^{(0)} \equiv -M_2^{1/2}\rho_1\boldsymbol{\mathscr{C}}_2/\rho && (p=0),\\
&\mathbf{a}_1^{(p)} \equiv S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1 && (p>0),\\
&\mathbf{a}_2^{(p)} \equiv 0 && (p>0),\\
&\mathbf{a}_1^{(-p)} \equiv 0 && (p>0),\\
&\mathbf{a}_2^{(-p)} \equiv S_{3/2}^{(p)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2 && (p>0),
\end{aligned} \tag{56}$$

in which:

$$S_m^{(n)}(x) = \sum_{p=0}^{n} \frac{(m+n)_{n-p}}{(p)!(n-p)!}(-x)^p = \sum_{p=0}^{n} \frac{(m+n)!}{(p)!(n-p)!(m+p)!}(-x)^p, \tag{57}$$

(with $S_m^{(0)}(x) = 1$ and $S_m^{(1)}(x) = m+1-x$) are numerical multiples (un-normalized) of the Sonine polynomials originally used in the Kinetic Theory of Gases by Burnett [16]. Here, as a consequence of Eq. (27) which must be satisfied, it follows directly that $\mathbf{a}_1^{(0)} = \mathbf{a}_2^{(0)}$. Also, as it is used in Chapman and Cowling (and in the present work), we further note the following orthogonality property of the Sonine polynomials:

$$\int_0^\infty \exp(-x)S_m^{(p)}(x)S_m^{(q)}(x)x^m\,dx = \{\Gamma(m+p+1)/p!\}\delta_{p,q}, \tag{58}$$

where $\delta_{p,q}$ is the Kronecker delta and $\Gamma(x)$ is the Gamma function. In Eq. (56) we note that Chapman and Cowling express the functions $\mathbf{a}_1^{(\pm p)}$ and $\mathbf{a}_2^{(\pm p)}$ only in terms of $p>0$. This varies in pattern from the manner in which they define the analogous viscosity functions, $\mathbf{b}_1^{(\pm p)}$ and $\mathbf{b}_2^{(\pm p)}$, where both $p>0$ and $p<0$ are employed [1,18], and thus the definitions may be somewhat confusing if considered at the same time. Expressed in the same pattern as the corresponding viscosity functions, Eq. (56) may also be written as:

$$\begin{aligned}
&\mathbf{a}_1^{(p)} = \mathbf{a}_1^{(0)} \equiv M_1^{1/2}\rho_2\boldsymbol{\mathscr{C}}_1/\rho && (p=0),\\
&\mathbf{a}_2^{(-p)} = \mathbf{a}_2^{(0)} \equiv -M_2^{1/2}\rho_1\boldsymbol{\mathscr{C}}_2/\rho && (p=0),\\
&\mathbf{a}_1^{(p)} \equiv 0 && (p<0),\\
&\mathbf{a}_2^{(-p)} \equiv 0 && (p<0),\\
&\mathbf{a}_1^{(p)} \equiv S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1 && (p>0),\\
&\mathbf{a}_2^{(-p)} \equiv S_{3/2}^{(p)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2 && (p>0).
\end{aligned} \tag{59}$$

Now, in order to determine the expansion coefficients, a_p, in Eq. (54), we note that from Eq. (31) one may write:

$$\{\mathbf{A},\mathbf{a}^{(q)}\} = \alpha_q, \tag{60}$$

where:

$$n^2\alpha_q = \int f_1^{(0)}\left(\mathscr{C}_1^2 - \frac{5}{2}\right)\mathbf{C}_1\cdot\mathbf{a}_1^{(q)}\,d\mathbf{c}_1 + \int f_2^{(0)}\left(\mathscr{C}_2^2 - \frac{5}{2}\right)\mathbf{C}_2\cdot\mathbf{a}_2^{(q)}\,d\mathbf{c}_2. \tag{61}$$

Integrating Eq. (61), one finds that $\alpha_q = 0$ when $q \neq \pm 1$ and that:

$$\alpha_1 = -\frac{15}{4}\frac{n_1}{n^2}\left(\frac{2kT}{m_1}\right)^{1/2}, \qquad \alpha_{-1} = -\frac{15}{4}\frac{n_2}{n^2}\left(\frac{2kT}{m_2}\right)^{1/2}. \tag{62}$$

Similarly, in order to determine the expansion coefficients, d_p, in Eq. (55) we note that from Eq. (32) one may write:

$$\{\mathbf{D},\mathbf{a}^{(q)}\} = \delta_q, \tag{63}$$

where:

$$n^2\delta_q = x_1^{-1}\int f_1^{(0)}\mathbf{C}_1\cdot\mathbf{a}_1^{(q)}\,d\mathbf{c}_1 - x_2^{-1}\int f_2^{(0)}\mathbf{C}_2\cdot\mathbf{a}_2^{(q)}\,d\mathbf{c}_2. \tag{64}$$

Integrating Eq. (64), one finds that:

$$\delta_0 = \frac{3}{2n}\left(\frac{2kT}{m_0}\right)^{1/2}, \qquad \delta_q = 0\ (q\neq 0). \tag{65}$$

Combining Eqs. (55), (56), and (63)–(65) yields the system of equations:

$$\sum_{p=-\infty}^{+\infty} d_p a_{pq} = \delta_q \quad (q = 0, \pm 1, \pm 2, \ldots, \pm\infty), \tag{66}$$

where:

$$a_{pq} \equiv \{\mathbf{a}^{(p)},\mathbf{a}^{(q)}\} \equiv a_{qp}. \tag{67}$$

If values for a_{pq} are known, all of the d_p can be determined by solving the algebraic system of equations represented by Eq. (66).

Now, to determine the expansion coefficients, a_p, one substitutes Eq. (56) into Eq. (54). Since $\tilde{\mathbf{A}} = \mathbf{A} - \mathrm{k}_T \mathbf{D}$, one has that:

$$\{\tilde{\mathbf{A}}, \mathbf{a}^{(q)}\} = \{\mathbf{A}, \mathbf{a}^{(q)}\} - \mathrm{k}_T \{\mathbf{D}, \mathbf{a}^{(q)}\} = \alpha_q - \mathrm{k}_T \delta_q, \tag{68}$$

which, since $\delta_q = 0$ ($q \neq 0$), yields:

$$\sum_{\substack{p=-\infty \\ p\neq 0}}^{+\infty} a_p a_{pq} = \alpha_q \quad (q = \pm 1, \pm 2, \ldots, \pm\infty). \tag{69}$$

The system of equations represented by Eq. (69), together with Eq. (67), determine the expansion coefficients, a_p.

From the preceding development, it can be shown that:

$$\{\mathbf{D}, \mathbf{D}\} = d_0 \delta_0,$$
$$\{\mathbf{D}, \mathbf{A}\} = d_1 \alpha_1 + d_{-1} \alpha_{-1},$$
$$\{\tilde{\mathbf{A}}, \tilde{\mathbf{A}}\} = a_1 \alpha_1 + a_{-1} \alpha_{-1}. \tag{70}$$

Thus, for any given order of the approximation, m, we may be express the transport coefficients as:

$$D_{12} = \frac{1}{2} x_1 x_2 \left(\frac{2kT}{m_0}\right)^{1/2} d_0, \tag{71}$$

$$D_T = -\frac{5}{4} x_1 x_2 \left(\frac{2kT}{m_0}\right)^{1/2} \left(x_1 M_1^{-1/2} d_1 + x_2 M_2^{-1/2} d_{-1}\right), \tag{72}$$

$$\mathrm{k}_T = \frac{D_T}{D_{12}} = -\frac{5}{2} \left(x_1 M_1^{-1/2} d_1 + x_2 M_2^{-1/2} d_{-1}\right)/d_0, \tag{73}$$

and:

$$\lambda = -\frac{5}{4} kn \left(\frac{2kT}{m_0}\right)^{1/2} (x_1 M_1^{-1/2} a_1 + x_2 M_2^{-1/2} a_{-1}), \tag{74}$$

where d_{-1}, d_0, and d_1 are determined from Eq. (66), and a_{-1} and a_1 are determined from Eq. (69).

6. The bracket integrals

In order to complete the evaluation of the diffusion, thermal diffusion, and thermal conductivity coefficients it is necessary to determine the expansion coefficients, d_{-1}, d_0, d_1, a_{-1}, and a_1. To determine these quantities it is necessary to evaluate the bracket integrals defined in Eq. (34) for $\{\mathbf{a}^{(p)}, \mathbf{a}^{(q)}\}$. Hence, it is first necessary to be able to evaluate the square bracket integrals of Eqs. (35)–(37), specifically, $[\mathbf{a}_1^{(p)}, \mathbf{a}_1^{(q)}]_1$, $[\mathbf{a}_1^{(p)}, \mathbf{a}_1^{(q)}]_{12}$, and $[\mathbf{a}_1^{(p)}, \mathbf{a}_2^{(q)}]_{12}$. Completion of this task requires integration over all of the collision variables and, additionally, requires knowledge of the form of the intermolecular potential. However, the intermolecular potential affects only integrations over the variables, g and b, because they determine the scattering angle, χ. All six of the other integrals (there are eight total in the collision operator) can be evaluated in some manner without specific knowledge of the intermolecular potential.

The specific bracket integrals mentioned above that require evaluation are given explicitly in Chapman and Cowling [1] but how they have been identified is not readily apparent in the text which focuses on very broad generalized expressions that can be difficult to interpret. In practice, the needed bracket integrals are most readily determined by considering Eqs. (21) and (22) directly. If these equations are expressed as:

$$n_1^2 I_1(\mathbf{A}_1) + n_1 n_2 I_{12}(\mathbf{A}_1) + n_1 n_2 I_{12}(\mathbf{A}_2) = -f_1^{(0)} \left(\frac{5}{2} - \mathscr{C}_1^2\right) \mathbf{C}_1, \tag{75}$$

$$n_2^2 I_2(\mathbf{A}_2) + n_1 n_2 I_{21}(\mathbf{A}_2) + n_1 n_2 I_{21}(\mathbf{A}_1) = -f_2^{(0)} \left(\frac{5}{2} - \mathscr{C}_2^2\right) \mathbf{C}_2, \tag{76}$$

then one can insert Sonine polynomial approximations for the solutions, $\mathbf{A}_1$ and $\mathbf{A}_2$, can multiply through the equations with additional Sonine polynomials, and can then integrate the equations. By using the orthogonality of the Sonine polynomials as given in Eq. (58), this process will yield a set of simultaneous equations involving the matrix coefficients necessary to specify the solutions. The RHSs of the equations determine the constants, α_q, and the LHSs yield combinations of bracket integrals that correspond to the desired a_{pq} elements of the matrix used to determine the a_p expansion coefficients which, in turn, determine the thermal conductivity via Eq. (74).

The easiest way to follow this process is via a low-order example. If one considers the Sonine polynomials used in the definition of the $\mathbf{a}_1^{(p)}$ and $\mathbf{a}_2^{(-p)}$ Chapman–Enskog expansion vectors, $p = 1$ is the lowest order associated with thermal conductivity with $S_m^{(1)}(x) = m + 1 - x$, such that $\mathbf{a}_1^{(1)} \equiv S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1 = (\frac{5}{2} - \mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1$ and $\mathbf{a}_2^{(-1)} \equiv S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2 = (\frac{5}{2} - \mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2$. Using only this order of approximation for $\tilde{\mathbf{A}}_1$ and $\tilde{\mathbf{A}}_2$, i.e. assuming that $\tilde{\mathbf{A}}_1 = a_1 \mathbf{a}_1^{(1)} = a_1(\frac{5}{2} - \mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1$ and $\tilde{\mathbf{A}}_2 = a_{-1}\mathbf{a}_2^{(-1)} = a_{-1}(\frac{5}{2} - \mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2$, and using the fact that $\mathbf{A}_1 = \tilde{\mathbf{A}}_1 + \mathrm{k}_T \mathbf{D}_1$ and $\mathbf{A}_2 = \tilde{\mathbf{A}}_2 + \mathrm{k}_T \mathbf{D}_2$, one may express Eqs. (75) and (76) as:

$$\begin{aligned} &n_1^2 I_1(\tilde{\mathbf{A}}_1 + \mathrm{k}_T \mathbf{D}_1) + n_1 n_2 I_{12}(\tilde{\mathbf{A}}_1 + \mathrm{k}_T \mathbf{D}_1) + n_1 n_2 I_{12}(\tilde{\mathbf{A}}_2 + \mathrm{k}_T \mathbf{D}_2) \\ &\quad = n_1^2 I_1(\tilde{\mathbf{A}}_1) + n_1 n_2 I_{12}(\tilde{\mathbf{A}}_1) + n_1 n_2 I_{12}(\tilde{\mathbf{A}}_2) \\ &\qquad + \mathrm{k}_T\left[n_1^2 I_1(\mathbf{D}_1) + n_1 n_2 I_{12}(\mathbf{D}_1 + \mathbf{D}_2)\right] \\ &\quad = -f_1^{(0)}\left(\frac{5}{2} - \mathscr{C}_1^2\right)\mathbf{C}_1, \end{aligned} \tag{77}$$

and:

$$\begin{aligned} &n_2^2 I_2(\tilde{\mathbf{A}}_2 + \mathrm{k}_T \mathbf{D}_2) + n_1 n_2 I_{21}(\tilde{\mathbf{A}}_2 + \mathrm{k}_T \mathbf{D}_2) + n_1 n_2 I_{21}(\tilde{\mathbf{A}}_1 + \mathrm{k}_T \mathbf{D}_1) \\ &\quad = n_2^2 I_2(\tilde{\mathbf{A}}_2) + n_1 n_2 I_{21}(\tilde{\mathbf{A}}_2) + n_1 n_2 I_{21}(\tilde{\mathbf{A}}_1) \\ &\qquad + \mathrm{k}_T\left[n_2^2 I_2(\mathbf{D}_2) + n_1 n_2 I_{21}(\mathbf{D}_1 + \mathbf{D}_2)\right] \\ &\quad = -f_2^{(0)}\left(\frac{5}{2} - \mathscr{C}_2^2\right)\mathbf{C}_2, \end{aligned} \tag{78}$$

which, given Eqs. (21) and (22), may be written as:

$$\begin{aligned} &n_1^2 I_1(\tilde{\mathbf{A}}_1) + n_1 n_2 I_{12}(\tilde{\mathbf{A}}_1) + n_1 n_2 I_{12}(\tilde{\mathbf{A}}_2) \\ &\quad = -f_1^{(0)}\left(\frac{5}{2} - \mathscr{C}_1^2\right)\mathbf{C}_1 - \mathrm{k}_T x_1^{-1} f_1^{(0)}\mathbf{C}_1 \\ &\quad = -f_1^{(0)} S_{3/2}^{(1)}(\mathscr{C}_1^2)\mathbf{C}_1 - \mathrm{k}_T x_1^{-1} f_1^{(0)} S_{3/2}^{(0)}(\mathscr{C}_1^2)\mathbf{C}_1, \end{aligned} \tag{79}$$

and:

$$\begin{aligned} &n_2^2 I_2(\tilde{\mathbf{A}}_2) + n_1 n_2 I_{21}(\tilde{\mathbf{A}}_2) + n_1 n_2 I_{21}(\tilde{\mathbf{A}}_1) \\ &\quad = -f_2^{(0)}\left(\frac{5}{2} - \mathscr{C}_2^2\right)\mathbf{C}_2 + \mathrm{k}_T x_2^{-1} f_2^{(0)}\mathbf{C}_2 \\ &\quad = -f_2^{(0)} S_{3/2}^{(1)}(\mathscr{C}_2^2)\mathbf{C}_2 + \mathrm{k}_T x_2^{-1} f_2^{(0)} S_{3/2}^{(0)}(\mathscr{C}_2^2)\mathbf{C}_2. \end{aligned} \tag{80}$$

Since in the current example we are limiting the discussion to the first-order expansion, Eqs. (79) and (80) may be written as:

$$\begin{aligned} &n_1^2 I_1\left(a_1 S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\right) + n_1 n_2 I_{12}\left(a_1 S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\right) \\ &\qquad + n_1 n_2 I_{12}\left(a_{-1} S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\right) \\ &\quad = -f_1^{(0)} S_{3/2}^{(1)}(\mathscr{C}_1^2)\mathbf{C}_1 - \mathrm{k}_T x_1^{-1} f_1^{(0)} S_{3/2}^{(0)}(\mathscr{C}_1^2)\mathbf{C}_1, \end{aligned} \tag{81}$$

and:

$$n_2^2 I_2\big(a_{-1} S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big) + n_1 n_2 I_{21}\big(a_{-1} S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big) + n_1 n_2 I_{21}\big(a_1 S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big) = -f_2^{(0)} S_{3/2}^{(1)}(\mathscr{C}_2^2)\mathbf{C}_2 + \mathrm{k}_T x_2^{-1} f_2^{(0)} S_{3/2}^{(0)}(\mathscr{C}_2^2)\mathbf{C}_2. \tag{82}$$

Now, multiplying through Eq. (81) by $S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1$ and through Eq. (82) by $S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2$ and integrating throughout them in the manner of Eqs. (12) and (13) for I_i and I_{ij} (with division by n^2), one has:

$$x_1^2 \int S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1 \cdot I_1\big(S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big)\,\mathrm{d}\mathbf{c}_1\, a_1 + x_1 x_2 \int S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1 \cdot I_{12}\big(S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big)\,\mathrm{d}\mathbf{c}_1 a_1 + x_1 x_2 \int S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1 \cdot I_{12}\big(S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big)\,\mathrm{d}\mathbf{c}_1 a_{-1} = -n^{-2} \int f_1^{(0)} S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1 \cdot S_{3/2}^{(1)}(\mathscr{C}_1^2)\mathbf{C}_1\,\mathrm{d}\mathbf{c}_1 - \mathrm{k}_T x_1^{-1} n^{-2} \int f_1^{(0)} S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1 \cdot S_{3/2}^{(0)}(\mathscr{C}_1^2)\mathbf{C}_1\,\mathrm{d}\mathbf{c}_1, \tag{83}$$

and:

$$x_2^2 \int S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2 \cdot I_2\big(S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big)\,\mathrm{d}\mathbf{c}_2 a_{-1} + x_1 x_2 \int S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2 \cdot I_{21}\big(S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big)\,\mathrm{d}\mathbf{c}_2 a_{-1} + x_1 x_2 \int S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2 \cdot I_{21}\big(S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big)\,\mathrm{d}\mathbf{c}_2 a_1 = -n^{-2} \int f_2^{(0)} S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2 \cdot S_{3/2}^{(1)}(\mathscr{C}_2^2)\mathbf{C}_2\,\mathrm{d}\mathbf{c}_2 + \mathrm{k}_T x_2^{-1} n^{-2} \int f_2^{(0)} S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2 \cdot S_{3/2}^{(0)}(\mathscr{C}_2^2)\mathbf{C}_2\,\mathrm{d}\mathbf{c}_2, \tag{84}$$

which, after employing the orthogonality property of Eq. (58) to eliminate the terms involving k_T, are expressible in bracket integral notation as:

$$x_1^2\big[S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_1 a_1 + x_1 x_2\big[S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_{12} a_1 + x_1 x_2\big[S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_{12} a_{-1} = \alpha_1, \tag{85}$$

and:

$$x_2^2\big[S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_2 a_{-1} + x_1 x_2\big[S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_{21} a_{-1} + x_1 x_2\big[S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_{21} a_1 = \alpha_{-1}. \tag{86}$$

Given the way that the solution has been approached and the low order of this example, it is readily apparent that these equations must be equivalent to:

$$a_{11} a_1 + a_{1-1} a_{-1} = \alpha_1, \tag{87}$$

$$a_{-11} a_1 + a_{-1-1} a_{-1} = \alpha_{-1}, \tag{88}$$

where:

$$a_{11} = x_1^2\big[S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_1 + x_1 x_2\big[S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_{12}, \tag{89}$$

$$a_{1-1} = x_1 x_2\big[S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_{12}, \tag{90}$$

$$a_{-11} = x_1 x_2\big[S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(1)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_{21}, \tag{91}$$

$$a_{-1-1} = x_2^2\big[S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_2 + x_1 x_2\big[S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(1)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_{21}, \tag{92}$$

and the integrals on the RHSs have been replaced with the appropriate α_q constants which, again due to the orthogonality of the Sonine polynomials, are defined only in the current example where $q = \pm 1$ and are otherwise zero. Eqs. (87) and (88) can, of course, be rearranged to cast them into the ordered form that corresponds directly to the form that the more general, higher-order problem has been expressed in, i.e.:

$$\begin{bmatrix} a_{-1-1} & a_{-11} \\ a_{1-1} & a_{11} \end{bmatrix} \begin{bmatrix} a_{-1} \\ a_1 \end{bmatrix} = \begin{bmatrix} \alpha_{-1} \\ \alpha_1 \end{bmatrix}. \tag{93}$$

From this simple example, generalization to higher-order expansions then is straightforward and one has for the relevant matrix elements:

$$a_{pq} = x_1^2\big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_1 + x_1 x_2\big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_{12}, \tag{94}$$

$$a_{p-q} = x_1 x_2\big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_{12}, \tag{95}$$

$$a_{-pq} = x_1 x_2\big[S_{3/2}^{(p)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_{21}, \tag{96}$$

$$a_{-p-q} = x_2^2\big[S_{3/2}^{(p)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(q)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_2 + x_1 x_2\big[S_{3/2}^{(p)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2, S_{3/2}^{(q)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_{21}, \tag{97}$$

where, for thermal conductivity, $p, q = 1, 2, 3, \ldots, m$.

One could also analyze a simple case of diffusion in a similar manner. For diffusion, one would obtain the following simple, first-order, matrix equation analogous to Eq. (93):

$$\begin{bmatrix} a_{-1-1} & a_{-10} & a_{-11} \\ a_{0-1} & a_{00} & a_{01} \\ a_{1-1} & a_{10} & a_{11} \end{bmatrix} \begin{bmatrix} d_{-1} \\ d_0 \\ d_1 \end{bmatrix} = \begin{bmatrix} 0 \\ \delta_0 \\ 0 \end{bmatrix}, \tag{98}$$

where the matrix elements a_{pq} are defined in exactly the same way as in Eqs. (94)–(97) with the only difference being that, for diffusion, $p, q = 0, 1, 2, 3, \ldots, m$. From the definitions of I_i and I_{ij} in Eqs. (12) and (13), it follows that Eqs. (96) and (97) are essentially identical to Eqs. (95) and (94), respectively, with the only difference being the interchange of the subscripts 1 and 2 representing the different components of the mixture. Thus, in general, the complete Chapman–Enskog solutions for diffusion and thermal conductivity require only the bracket integrals:

$$\big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_1, \tag{99}$$

$$\big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_{12}, \tag{100}$$

$$\big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_{12}, \tag{101}$$

which are exactly those identified by Chapman and Cowling. Some further simplification of the problem can be obtained by noting from Chapman and Cowling that, in the limit of a simple (single) gas where $m_1 = m_2$, $n_1 = n_2 = n$, and $\alpha_{21} = \alpha_{12} = \alpha_1$, one has that $[F, G]_1 = [F_1, G_1 + G_2]_{12} = ([F_1, G_1]_{12} + [F_1, G_2]_{12})$ which, in the current problem, equates to:

$$\big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_1 = \Big(\big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1\big]_{12} + \big[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2\big]_{12}\Big)\Big|_{\substack{m_1 = m_2 \\ n_1 = n_2 = n \\ \alpha_{12} = \alpha_{21} = \alpha_1}}. \tag{102}$$

At this point, it still remains to perform the six integrations unrelated to the intermolecular potential model that is employed in order to complete the evaluation of the two necessary bracket integrals on the RHS of Eq. (102). For the relevant details of this integral evaluation process, one should refer to the text of Chapman and Cowling [1]. We note that Chapman and Cowling make use of the following definition of the Sonine polynomials:

$$\left(\frac{\mathrm{S}}{s}\right)^{m+1} \exp(-x\mathrm{S}) \equiv (1-s)^{-m-1} \exp\left(\frac{-xs}{1-s}\right) = \sum_{n=0}^{\infty} s^n S_m^{(n)}(x), \tag{103}$$

where $\mathrm{S} = s/(1-s)$ and, likewise, $\mathrm{T} = t/(1-t)$, to express the needed bracket integrals in terms of the coefficients of expansions in the arbitrarily introduced variables, s and t. Thus, it is possible after following Chapman and Cowling to determine that:

$$[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2]_{12} = \mathrm{coeff}[s^p t^q] \quad \text{in} \quad \left(\frac{\mathrm{ST}}{st}\right)^{5/2} \pi^{-3} \iiint \{H_{12}(0) - H_{12}(\chi)\} g b \,\mathrm{d}b \,\mathrm{d}\varepsilon \,\mathrm{d}\boldsymbol{g}, \tag{104}$$

and:

$$[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1]_{12} = \mathrm{coeff}[s^p t^q] \quad \text{in} \quad \left(\frac{\mathrm{ST}}{st}\right)^{5/2} \pi^{-3} \iiint \{H_1(0) - H_1(\chi)\} g b \,\mathrm{d}b \,\mathrm{d}\varepsilon \,\mathrm{d}\boldsymbol{g}, \tag{105}$$

where $\boldsymbol{g} \equiv (m_0 M_1 M_2/2kT)^{1/2}\mathbf{g}_{21}$. Note that the retention of a single g in the integrands of Eqs. (104) and (105) (as opposed to g) is not a typographical error but, rather, is the exact notation used by Chapman and Cowling. After integration over ε and the directions of $\boldsymbol{g}$ (which changes the constants somewhat), one can express the χ-dependent portions of the RHS bracketed integrals of Eqs. (104) and (105) as:

$$\begin{aligned}
&\left(\frac{\mathrm{ST}}{st}\right)^{5/2} (M_1M_2)^{-1/2}\pi^{-3/2} H_{12}(\chi) \\
&\quad = \exp(-g^2) \sum_r \sum_n \{2M_1M_2 st[1-\cos(\chi)]\}^r \\
&\qquad \times \left(\frac{g^{2r}}{r!}\right)(M_2 s + M_1 t)^n \{(n+1)S_{r+1/2}^{(n+1)}(g^2) \\
&\qquad + [1-\cos(\chi)] g^2 S_{r+3/2}^{(n)}(g^2)\},
\end{aligned} \tag{106}$$

and:

$$\begin{aligned}
&\left(\frac{\mathrm{ST}}{st}\right)^{5/2} \pi^{-3/2} H_1(\chi) = \exp(-g^2) \\
&\quad \times \sum_r \sum_n \{st[M_1^2 + M_2^2 + 2M_1M_2\cos(\chi)]\}^r \\
&\quad \times \left(\frac{g^{2r}}{r!}\right)\{M_2(s+t) - (M_2 - M_1)st\}^n \{M_1(n+1)S_{r+1/2}^{(n+1)}(g^2) \\
&\quad + [M_1 + M_2\cos(\chi)] g^2 S_{r+3/2}^{(n)}(g^2)\}.
\end{aligned} \tag{107}$$

In both of these cases, the coefficient of $[s^p t^q]$ yields a polynomial in powers of g^2 and $\cos(\chi)$ that is multiplied by $\exp(-g^2)$ and in which each term is some function of the molecular masses via M_1 and M_2. The χ-independent portions of the RHS bracketed integrals of Eqs. (104) and (105) are obtained by the simple expedient of setting $\chi = 0$ in Eqs. (106) and (107) which yields overall terms in the combined polynomials involving $[1 - \cos^\ell(\chi)]$. Thus, it is possible to express Eqs. (104) and (105) as:

$$[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_2^2)\boldsymbol{\mathscr{C}}_2]_{12} = 8M_2^{p+1/2}M_1^{q+1/2}\sum_{r,\ell} A_{pqr\ell}\Omega_{12}^{(\ell)}(r), \tag{108}$$

and:

$$[S_{3/2}^{(p)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1, S_{3/2}^{(q)}(\mathscr{C}_1^2)\boldsymbol{\mathscr{C}}_1]_{12} = 8\sum_{r,\ell} A'_{pqr\ell}\Omega_{12}^{(\ell)}(r), \tag{109}$$

where the omega integrals are defined as:

$$\Omega_{12}^{(\ell)}(r) \equiv \left(\frac{kT}{2\pi m_0 M_1 M_2}\right)^{1/2} \int_0^\infty \exp(-g^2) g^{(2r+3)} \phi_{12}^{(\ell)} \,\mathrm{d}g, \tag{110}$$

with:

$$\phi_{12}^{(\ell)} \equiv 2\pi \int_0^\pi [1-\cos^\ell(\chi)] b \,\mathrm{d}b. \tag{111}$$

As stated by Chapman and Cowling [1]:

> "Explicit expressions for $[A_{pqr\ell}$ and $A'_{pqr\ell}]$ can be obtained from [Eqs. (106) and (107)] using [Eq. (57)] for $S_m^{(n)}(x)$. In view of the complication of these expressions it is, however, better in practice to calculate any desired values of $[A_{pqr\ell}$ and $A'_{pqr\ell}]$ directly from [Eqs. (106) and (107)]."

We have explored this approach as suggested by Chapman and Cowling using *Mathematica*® up to order 70 and are reporting our order 70 diffusion and thermal conductivity results here. Note that in all of our expressions, we have retained fully the general dependence of the expressions on the mole fractions, the molecular masses, and the models of the intermolecular potential that can be employed. For example, from Eqs. (66) and (69), since there exists Eq. (67) symmetry in the off-diagonal elements such that $a_{pq} = a_{qp}$, one can readily generate the matrix equations of Eqs. (93) and (98) for the order 1 diffusion and thermal conductivity solutions. For these first-order solutions, we have explicitly determined the general expressions for the a_{pq} matrix elements to be:

$$\begin{aligned}
a_{-1-1} &= x_2^2\big(4\Omega_2^{(2)}(2)\big) + x_1x_2\big(10\big(5M_1^3 + 6M_1M_2^2\big)\Omega_{12}^{(1)}(1) \\
&\quad - 40M_1^3\Omega_{12}^{(1)}(2) + 8M_1^3\Omega_{12}^{(1)}(3) + 16M_1^2M_2\Omega_{12}^{(2)}(2)\big),
\end{aligned} \tag{112}$$

$$a_{-10} = x_1x_2\big(-20M_1^2M_2^{1/2}\Omega_{12}^{(1)}(1) + 8M_1^2M_2^{1/2}\Omega_{12}^{(1)}(2)\big), \tag{113}$$

$$\begin{aligned}
a_{-11} &= x_1x_2\big(-110M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(1) + 40M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(2) \\
&\quad - 8M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(3) + 16M_1^{3/2}M_2^{3/2}\Omega_{12}^{(2)}(2)\big),
\end{aligned} \tag{114}$$

$$a_{0-1} = x_1x_2\big(-20M_1^2M_2^{1/2}\Omega_{12}^{(1)}(1) + 8M_1^2M_2^{1/2}\Omega_{12}^{(1)}(2)\big), \tag{115}$$

$$a_{00} = x_1x_2\big(8M_1M_2\Omega_{12}^{(1)}(1)\big), \tag{116}$$

$$a_{01} = x_1x_2\big(20M_1^{1/2}M_2^2\Omega_{12}^{(1)}(1) - 8M_1^{1/2}M_2^2\Omega_{12}^{(1)}(2)\big), \tag{117}$$

$$\begin{aligned}
a_{1-1} &= x_1x_2\big(-110M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(1) + 40M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(2) \\
&\quad - 8M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(3) + 16M_1^{3/2}M_2^{3/2}\Omega_{12}^{(2)}(2)\big),
\end{aligned} \tag{118}$$

$$a_{10} = x_1x_2\big(20M_1^{1/2}M_2^2\Omega_{12}^{(1)}(1) - 8M_1^{1/2}M_2^2\Omega_{12}^{(1)}(2)\big), \tag{119}$$

$$\begin{aligned}
a_{11} &= x_1^2\big(4\Omega_1^{(2)}(2)\big) + x_1x_2\big(10\big(6M_1^2M_2 + 5M_2^3\big)\Omega_{12}^{(1)}(1) \\
&\quad - 40M_2^3\Omega_{12}^{(1)}(2) + 8M_2^3\Omega_{12}^{(1)}(3) + 16M_1M_2^2\Omega_{12}^{(2)}(2)\big).
\end{aligned} \tag{120}$$

The symmetry in the off-diagonal elements is obvious, with $a_{-10} = a_{0-1}$, $a_{-11} = a_{1-1}$, and $a_{01} = a_{10}$. Likewise, for order 2, Eqs. (66) and (69) plus symmetry give the following matrix equations for diffusion and thermal conductivity, respectively:

$$\begin{bmatrix}
a_{-2-2} & a_{-2-1} & a_{-20} & a_{-21} & a_{-22} \\
a_{-1-2} & a_{-1-1} & a_{-10} & a_{-11} & a_{-12} \\
a_{0-2} & a_{0-1} & a_{00} & a_{01} & a_{02} \\
a_{1-2} & a_{1-1} & a_{10} & a_{11} & a_{12} \\
a_{2-2} & a_{2-1} & a_{20} & a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix} d_{-2} \\ d_{-1} \\ d_0 \\ d_1 \\ d_2 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ \delta_0 \\ 0 \\ 0 \end{bmatrix}, \tag{121}$$

$$\begin{bmatrix} a_{-2-2} & a_{-2-1} & a_{-21} & a_{-22} \\ a_{-1-2} & a_{-1-1} & a_{-11} & a_{-12} \\ a_{1-2} & a_{1-1} & a_{11} & a_{12} \\ a_{2-2} & a_{2-1} & a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} a_{-2} \\ a_{-1} \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \alpha_{-1} \\ \alpha_1 \\ 0 \end{bmatrix}, \tag{122}$$

and we have determined the general expressions for the a_{pq} matrix elements to be:

$$\begin{aligned} a_{-2-2} = {} & x_2^2\left(\frac{77}{4}\Omega_2^{(2)}(2) - 7\Omega_2^{(2)}(3) + \Omega_2^{(2)}(4)\right) \\ & + x_1x_2\left(\frac{35}{8}\left(35M_1^5 + 168M_1^3M_2^2 + 40M_1M_2^4\right)\Omega_{12}^{(1)}(1)\right. \\ & - 49\left(5M_1^5 + 12M_1^3M_2^2\right)\Omega_{12}^{(1)}(2) \\ & + \left(133M_1^5 + 108M_1^3M_2^2\right)\Omega_{12}^{(1)}(3) - 28M_1^5\Omega_{12}^{(1)}(4) \\ & + 2M_1^5\Omega_{12}^{(1)}(5) + 28\left(7M_1^4M_2 + 4M_1^2M_2^3\right)\Omega_{12}^{(2)}(2) \\ & - 112M_1^4M_2\Omega_{12}^{(2)}(3) + 16M_1^4M_2\Omega_{12}^{(2)}(4) \\ & \left. + 16M_1^3M_2^2\Omega_{12}^{(3)}(3)\right), \end{aligned} \tag{123}$$

$$\begin{aligned} a_{-2-1} = a_{-1-2} = {} & x_2^2\left(7\Omega_2^{(2)}(2) - 2\Omega_2^{(2)}(3)\right) \\ & + x_1x_2\left(\frac{35}{2}\left(5M_1^4 + 12M_1^2M_2^2\right)\Omega_{12}^{(1)}(1)\right. \\ & - 21\left(5M_1^4 + 4M_1^2M_2^2\right)\Omega_{12}^{(1)}(2) + 38M_1^4\Omega_{12}^{(1)}(3) \\ & - 4M_1^4\Omega_{12}^{(1)}(4) + 56M_1^3M_2\Omega_{12}^{(2)}(2) \\ & \left. - 16M_1^3M_2\Omega_{12}^{(2)}(3)\right), \end{aligned} \tag{124}$$

$$\begin{aligned} a_{-20} = a_{0-2} = {} & x_1x_2\left(-35M_1^3M_2^{1/2}\Omega_{12}^{(1)}(1)\right. \\ & \left. + 28M_1^3M_2^{1/2}\Omega_{12}^{(1)}(2) - 4M_1^3M_2^{1/2}\Omega_{12}^{(1)}(3)\right), \end{aligned} \tag{125}$$

$$\begin{aligned} a_{-21} = a_{1-2} = {} & x_1x_2\left(-\frac{595}{2}M_1^{5/2}M_2^{3/2}\Omega_{12}^{(1)}(1)\right. \\ & + 189M_1^{5/2}M_2^{3/2}\Omega_{12}^{(1)}(2) - 38M_1^{5/2}M_2^{3/2}\Omega_{12}^{(1)}(3) \\ & + 4M_1^{5/2}M_2^{3/2}\Omega_{12}^{(1)}(4) + 56M_1^{5/2}M_2^{3/2}\Omega_{12}^{(2)}(2) \\ & \left. - 16M_1^{5/2}M_2^{3/2}\Omega_{12}^{(2)}(3)\right), \end{aligned} \tag{126}$$

$$\begin{aligned} a_{-22} = a_{2-2} = {} & x_1x_2\left(-\frac{8505}{8}M_1^{5/2}M_2^{5/2}\Omega_{12}^{(1)}(1)\right. \\ & + 833M_1^{5/2}M_2^{5/2}\Omega_{12}^{(1)}(2) - 241M_1^{5/2}M_2^{5/2}\Omega_{12}^{(1)}(3) \\ & + 28M_1^{5/2}M_2^{5/2}\Omega_{12}^{(1)}(4) - 2M_1^{5/2}M_2^{5/2}\Omega_{12}^{(1)}(5) \\ & + 308M_1^{5/2}M_2^{5/2}\Omega_{12}^{(2)}(2) - 112M_1^{5/2}M_2^{5/2}\Omega_{12}^{(2)}(3) \\ & \left. + 16M_1^{5/2}M_2^{5/2}\Omega_{12}^{(2)}(4) - 16M_1^{5/2}M_2^{5/2}\Omega_{12}^{(3)}(3)\right), \end{aligned} \tag{127}$$

$$\begin{aligned} a_{-1-1} = {} & x_2^2\left(4\Omega_2^{(2)}(2)\right) + x_1x_2\left(10\left(5M_1^3 + 6M_1M_2^2\right)\Omega_{12}^{(1)}(1)\right. \\ & \left. - 40M_1^3\Omega_{12}^{(1)}(2) + 8M_1^3\Omega_{12}^{(1)}(3) + 16M_1^2M_2\Omega_{12}^{(2)}(2)\right), \end{aligned} \tag{128}$$

$$\begin{aligned} a_{-10} = a_{0-1} = {} & x_1x_2\left(-20M_1^2M_2^{1/2}\Omega_{12}^{(1)}(1)\right. \\ & \left. + 8M_1^2M_2^{1/2}\Omega_{12}^{(1)}(2)\right), \end{aligned} \tag{129}$$

$$\begin{aligned} a_{-11} = a_{1-1} = {} & x_1x_2\left(-110M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(1) + 40M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(2)\right. \\ & \left. - 8M_1^{3/2}M_2^{3/2}\Omega_{12}^{(1)}(3) + 16M_1^{3/2}M_2^{3/2}\Omega_{12}^{(2)}(2)\right), \end{aligned} \tag{130}$$

$$\begin{aligned} a_{-12} = a_{2-1} = {} & x_1x_2\left(-\frac{595}{2}M_1^{3/2}M_2^{5/2}\Omega_{12}^{(1)}(1)\right. \\ & + 189M_1^{3/2}M_2^{5/2}\Omega_{12}^{(1)}(2) - 38M_1^{3/2}M_2^{5/2}\Omega_{12}^{(1)}(3) \\ & + 4M_1^{3/2}M_2^{5/2}\Omega_{12}^{(1)}(4) + 56M_1^{3/2}M_2^{5/2}\Omega_{12}^{(2)}(2) \\ & \left. - 16M_1^{3/2}M_2^{5/2}\Omega_{12}^{(2)}(3)\right), \end{aligned} \tag{131}$$

$$a_{00} = x_1x_2\left(8M_1M_2\Omega_{12}^{(1)}(1)\right), \tag{132}$$

$$a_{01} = a_{10} = x_1x_2\left(20M_1^{1/2}M_2^2\Omega_{12}^{(1)}(1) - 8M_1^{1/2}M_2^2\Omega_{12}^{(1)}(2)\right), \tag{133}$$

$$\begin{aligned} a_{02} = a_{20} = {} & x_1x_2\left(35M_1^{1/2}M_2^3\Omega_{12}^{(1)}(1)\right. \\ & \left. - 28M_1^{1/2}M_2^3\Omega_{12}^{(1)}(2) + 4M_1^{1/2}M_2^3\Omega_{12}^{(1)}(3)\right), \end{aligned} \tag{134}$$

$$\begin{aligned} a_{11} = {} & x_1^2\left(4\Omega_1^{(2)}(2)\right) + x_1x_2\left(10\left(6M_1^2M_2 + 5M_2^3\right)\Omega_{12}^{(1)}(1)\right. \\ & \left. + 8M_2^3\Omega_{12}^{(1)}(3) - 40M_2^3\Omega_{12}^{(1)}(2) + 16M_1M_2^2\Omega_{12}^{(2)}(2)\right), \end{aligned} \tag{135}$$

$$\begin{aligned} a_{12} = a_{21} = {} & x_1^2\left(7\Omega_1^{(2)}(2) - 2\Omega_1^{(2)}(3)\right) \\ & + x_1x_2\left(\frac{35}{2}\left(12M_1^2M_2^2 + 5M_2^4\right)\Omega_{12}^{(1)}(1)\right. \\ & - 21\left(4M_1^2M_2^2 + 5M_2^4\right)\Omega_{12}^{(1)}(2) + 38M_2^4\Omega_{12}^{(1)}(3) \\ & - 4M_2^4\Omega_{12}^{(1)}(4) + 56M_1M_2^3\Omega_{12}^{(2)}(2) \\ & \left. - 16M_1M_2^3\Omega_{12}^{(2)}(3)\right), \end{aligned} \tag{136}$$

$$\begin{aligned} a_{22} = {} & x_1^2\left(\frac{77}{4}\Omega_1^{(2)}(2) - 7\Omega_1^{(2)}(3) + \Omega_1^{(2)}(4)\right) \\ & + x_1x_2\left(\frac{35}{8}\left(40M_1^4M_2 + 168M_1^2M_2^3 + 35M_2^5\right)\Omega_{12}^{(1)}(1)\right. \\ & - 49\left(12M_1^2M_2^3 + 5M_2^5\right)\Omega_{12}^{(1)}(2) \\ & + \left(108M_1^2M_2^3 + 133M_2^5\right)\Omega_{12}^{(1)}(3) - 28M_2^5\Omega_{12}^{(1)}(4) \\ & + 2M_2^5\Omega_{12}^{(1)}(5) + 28\left(4M_1^3M_2^2 + 7M_1M_2^4\right)\Omega_{12}^{(2)}(2) \\ & - 112M_1M_2^4\Omega_{12}^{(2)}(3) + 16M_1M_2^4\Omega_{12}^{(2)}(4) \\ & \left. + 16M_1^2M_2^3\Omega_{12}^{(3)}(3)\right). \end{aligned} \tag{137}$$

In Eqs. (123)–(137), as opposed to Eqs. (112)–(120), the symmetry has been expressed explicitly. While we could, certainly, go even higher in order detailing explicit expressions for the matrix elements, the expressions rapidly become unwieldy in analytical form. If one wishes to verify our order 2 expressions or if additional explicit expressions for the order 3 results are desired, we note that they can be determined from the relevant expressions which have been reported previously in the literature [2,3,21,22]. In going to higher orders than those available in the current literature, as has been done in this work, it is clear from a comparison of the order 1 and order 2 expressions above that all previously generated expressions are retained at each higher order and need not be recomputed. Most importantly in this work, regardless of the order of the expansion used, the dependencies of the matrix elements on mole fractions and molecular masses are carried through in general form. Molecular diameters and the specific intermolecular potential model used are also carried through in general form via the omega integrals which may also be defined in terms of the quantity, σ_{12}. This quantity is, in purely general terms, only a convenient, arbitrarily chosen length in the range of b. Thus, it is often convenient to express the omega integrals as [1]:

$$\Omega_{12}^{(\ell)}(r) = \frac{1}{2}\pi\sigma_{12}^2\left(\frac{2\pi kT}{m_0M_1M_2}\right)^{1/2}W_{12}^{(\ell)}(r), \tag{138}$$

where:

$$W_{12}^{(\ell)}(r) \equiv \int_0^\infty \exp(-g^2) g^{2r+2} \times \int_0^\pi [1-\cos^\ell(\chi)](b/\sigma_{12})\, \mathrm{d}(b/\sigma_{12})\, \mathrm{d}(g^2) = 2\int_0^\infty \exp(-g^2) g^{2r+3} \times \int_0^\pi [1-\cos^\ell(\chi)](b/\sigma_{12})\, \mathrm{d}(b/\sigma_{12})\, \mathrm{d}g, \tag{139}$$

and where the corresponding simple gas expressions are:

$$\Omega_1^{(\ell)}(r) = \sigma_1^2 \left(\frac{\pi k T}{m_1}\right)^{1/2} W_1^{(\ell)}(r), \tag{140}$$

with:

$$W_1^{(\ell)}(r) = 2\int_0^\infty \exp(-g^2) g^{(2r+3)} \times \int_0^\pi [1-\cos^\ell(\chi)](b/\sigma_1)\, \mathrm{d}(b/\sigma_1)\, \mathrm{d}g, \tag{141}$$

and:

$$\Omega_2^{(\ell)}(r) = \sigma_2^2 \left(\frac{\pi k T}{m_2}\right)^{1/2} W_2^{(\ell)}(r), \tag{142}$$

with:

$$W_2^{(\ell)}(r) = 2\int_0^\infty \exp(-g^2) g^{(2r+3)} \times \int_0^\pi [1-\cos^\ell(\chi)](b/\sigma_2)\, \mathrm{d}(b/\sigma_2)\, \mathrm{d}g. \tag{143}$$

In Eqs. (138)–(143), σ_1 and σ_2 are arbitrary scale lengths associated with collisions between like molecules of type 1 or type 2, respectively, while σ_{12} is associated with collisions between unlike molecules of types 1 and 2. These scale lengths are commonly associated with some concept of the molecular diameters depending upon the specific details of the intermolecular potential model that is employed. In the current work, we are reporting results for the case of rigid-sphere molecules because the form of the rigid-sphere potential model allows all of the omega integrals to be evaluated analytically eliminating the need for any numerical integrations in the current work. Some of the details of this model are described in the following section.

7. The case of rigid-sphere molecules

For the specific case of a binary, rigid-sphere, gas mixture, one has an intermolecular potential model of the form [20]:

$$U(r) = \begin{cases} \infty, & r \leqslant \sigma_{12}, \\ 0, & r > \sigma_{12}, \end{cases} \tag{144}$$

where $\sigma_{12} = \frac{1}{2}(\sigma_1 + \sigma_2)$, and σ_1 and σ_2 are the actual diameters of the colliding spherical molecules. Under the rigid-sphere assumption, one then has the collisional relationships $b = \sigma_{12}\cos(\frac{1}{2}\chi)$ and $b\,\mathrm{d}b = -\frac{1}{4}\sigma_{12}^2 \sin(\chi)\,\mathrm{d}\chi$. Using these in Eq. (139) yields:

$$W_{12}^{(\ell)}(r) = \frac{1}{4}\left[2 - \frac{1}{\ell+1}\left(1+(-1)^\ell\right)\right](r+1)!, \tag{145}$$

such that:

$$\Omega_{12}^{(\ell)}(r) = \frac{1}{2}\sigma_{12}^2 \left(\frac{2\pi k T}{m_0 M_1 M_2}\right)^{1/2} \times \frac{1}{4}\left[2 - \frac{1}{\ell+1}\left(1+(-1)^\ell\right)\right](r+1)!, \tag{146}$$

and, since $W_1^{(\ell)}(r) = W_2^{(\ell)}(r) = W_{12}^{(\ell)}(r)$, the corresponding simple-gas expressions are:

$$\Omega_1^{(\ell)}(r) = \sigma_1^2 \left(\frac{\pi k T}{m_1}\right)^{1/2} \frac{1}{4}\left[2 - \frac{1}{\ell+1}\left(1+(-1)^\ell\right)\right](r+1)!, \tag{147}$$

and:

$$\Omega_2^{(\ell)}(r) = \sigma_2^2 \left(\frac{\pi k T}{m_2}\right)^{1/2} \frac{1}{4}\left[2 - \frac{1}{\ell+1}\left(1+(-1)^\ell\right)\right](r+1)!. \tag{148}$$

All of these omega integrals are readily evaluated for the purpose of the current work and, thus, one has the following simplified, rigid-sphere, matrix elements for the order 2 diffusion and thermal conductivity approximations:

$$a_{-2-2} = x_2^2\left\{\sqrt{\frac{\pi k T}{m_2}}\left(\frac{45}{2}\right)\sigma_2^2\right\} + x_1 x_2\left\{\sqrt{\frac{2\pi k T}{m_0 M_1 M_2}}\left(\frac{433}{16}M_1^5 + 68 M_1^4 M_2 + \frac{459}{2}M_1^3 M_2^2 + 112 M_1^2 M_2^3 + \frac{175}{2}M_1 M_2^4\right)\sigma_{12}^2\right\}, \tag{149}$$

$$a_{-2-1} = a_{-1-2} = x_2^2\left\{-\sqrt{\frac{\pi k T}{m_2}}(2)\sigma_2^2\right\} + x_1 x_2\left\{-\sqrt{\frac{2\pi k T}{m_0 M_1 M_2}}\left(\frac{23}{4}M_1^4 + 8 M_1^3 M_2 + 21 M_1^2 M_2^2\right)\sigma_{12}^2\right\}, \tag{150}$$

$$a_{-20} = a_{0-2} = x_1 x_2\left\{\sqrt{\frac{2\pi k T}{m_0 M_1 M_2}}\left(\frac{1}{2}M_1^3 M_2^{1/2}\right)\sigma_{12}^2\right\}, \tag{151}$$

$$a_{-21} = a_{1-2} = x_1 x_2\left\{\sqrt{\frac{2\pi k T}{m_0 M_1 M_2}}\left(\frac{75}{4}M_1^{5/2} M_2^{3/2}\right)\sigma_{12}^2\right\}, \tag{152}$$

$$a_{-22} = a_{2-2} = x_1 x_2\left\{-\sqrt{\frac{2\pi k T}{m_0 M_1 M_2}}\left(\frac{2625}{16}M_1^{5/2} M_2^{5/2}\right)\sigma_{12}^2\right\}, \tag{153}$$

$$a_{-1-1} = x_2^2\left\{\sqrt{\frac{\pi k T}{m_2}}(8)\sigma_2^2\right\} + x_1 x_2\left\{\sqrt{\frac{2\pi k T}{m_0 M_1 M_2}}\left(13 M_1^3 + 16 M_1^2 M_2 + 30 M_1 M_2^2\right)\sigma_{12}^2\right\}, \tag{154}$$

$$a_{-10} = a_{0-1} = x_1 x_2\left\{\sqrt{\frac{2\pi k T}{m_0 M_1 M_2}}\left(2 M_1^2 M_2^{1/2}\right)\sigma_{12}^2\right\}, \tag{155}$$

$$a_{-11} = a_{1-1} = x_1 x_2\left\{-\sqrt{\frac{2\pi k T}{m_0 M_1 M_2}}\left(27 M_1^{3/2} M_2^{3/2}\right)\sigma_{12}^2\right\}, \tag{156}$$

$$a_{-12} = a_{2-1} = x_1 x_2 \left\{ \sqrt{\frac{2\pi kT}{m_0 M_1 M_2}} \left(\frac{75}{4} M_1^{3/2} M_2^{5/2} \right) \sigma_{12}^2 \right\}, \tag{157}$$

$$a_{00} = x_1 x_2 \left\{ \sqrt{\frac{2\pi kT}{m_0 M_1 M_2}} (4 M_1 M_2) \sigma_{12}^2 \right\}, \tag{158}$$

$$a_{01} = a_{10} = x_1 x_2 \left\{ \sqrt{\frac{2\pi kT}{m_0 M_1 M_2}} \left(2 M_1^{1/2} M_2^2\right) \sigma_{12}^2 \right\}, \tag{159}$$

$$a_{02} = a_{20} = x_1 x_2 \left\{ \sqrt{\frac{2\pi kT}{m_0 M_1 M_2}} \left(\frac{1}{2} M_1^{1/2} M_2^3 \right) \sigma_{12}^2 \right\}, \tag{160}$$

$$a_{11} = x_1^2 \left\{ \sqrt{\frac{\pi kT}{m_1}} (8) \sigma_1^2 \right\} + x_1 x_2 \left\{ \sqrt{\frac{2\pi kT}{m_0 M_1 M_2}} \left(13 M_2^3 + 16 M_1 M_2^2 + 30 M_1^2 M_2\right) \sigma_{12}^2 \right\}, \tag{161}$$

$$a_{12} = a_{21} = x_1^2 \left\{ -\sqrt{\frac{\pi kT}{m_1}} (2) \sigma_1^2 \right\} + x_1 x_2 \left\{ -\sqrt{\frac{2\pi kT}{m_0 M_1 M_2}} \left(\frac{23}{4} M_2^4 + 8 M_1 M_2^3 + 21 M_1^2 M_2^2 \right) \sigma_{12}^2 \right\}, \tag{162}$$

$$a_{22} = x_1^2 \left\{ \sqrt{\frac{\pi kT}{m_1}} \left(\frac{45}{2} \right) \sigma_1^2 \right\} + x_1 x_2 \left\{ \sqrt{\frac{2\pi kT}{m_0 M_1 M_2}} \left(\frac{433}{16} M_2^5 + 68 M_1 M_2^4 + \frac{459}{2} M_1^2 M_2^3 + 112 M_1^3 M_2^2 + \frac{175}{2} M_1^4 M_2 \right) \sigma_{12}^2 \right\}. \tag{163}$$

In our results, this methodology is continued to arbitrary order for rigid-sphere molecules with the full dependencies of the matrix elements on the molecular masses, mole fractions, and molecular diameters being retained explicitly up to the final point of actual evaluation via matrix inversion. As indicated previously, adaptation of this work to more realistic potential models is straightforward since the potential model is present in a fully general form in the bracket integral expressions via the omega integrals.

8. Results

The quantities of major interest in the present work are the diffusion coefficient, the thermal diffusion coefficient, the thermal-diffusion ratio, and the thermal conductivity coefficient given in Eqs. (71)–(74), respectively, as well as the Chapman–Enskog diffusion solutions:

$$\mathbb{D}_1(\mathscr{C}_1) = \sum_{p=0}^{+\infty} d_p S_{3/2}^{(p)}\left(\mathscr{C}_1^2\right) \mathscr{C}_1, \tag{164}$$

$$\mathbb{D}_2(\mathscr{C}_2) = \sum_{p=0}^{+\infty} d_{-p} S_{3/2}^{(p)}\left(\mathscr{C}_2^2\right) \mathscr{C}_2, \tag{165}$$

and the Chapman–Enskog thermal conductivity solutions:

$$\tilde{\mathbb{A}}_1(\mathscr{C}_1) = \sum_{p=1}^{+\infty} a_p S_{3/2}^{(p)}\left(\mathscr{C}_1^2\right) \mathscr{C}_1, \tag{166}$$

$$\tilde{\mathbb{A}}_2(\mathscr{C}_2) = \sum_{p=1}^{+\infty} a_{-p} S_{3/2}^{(p)}\left(\mathscr{C}_2^2\right) \mathscr{C}_2, \tag{167}$$

in which the expansion coefficients, $d_{\pm p}$ and $a_{\pm p}$ have the dimensions of length.

Of additional interest, we note that d_0 is actually an integral on either $\mathbb{D}_1$ or $\mathbb{D}_2$ and may be expressed as:

$$d_0 = \frac{\rho\rho_2}{M_1^{1/2}} \frac{8}{3\sqrt{\pi}} \int_0^\infty \exp\left(-\mathscr{C}_1^2\right) \mathscr{C}_1^3 \mathbb{D}_1(\mathscr{C}_1)\, \mathrm{d}\mathscr{C}_1 = \frac{\rho\rho_1}{M_2^{1/2}} \frac{8}{3\sqrt{\pi}} \int_0^\infty \exp\left(-\mathscr{C}_2^2\right) \mathscr{C}_2^3 \mathbb{D}_2(\mathscr{C}_2)\, \mathrm{d}\mathscr{C}_2, \tag{168}$$

while d_1 and d_{-1} are also integrals on $\mathbb{D}_1$ and $\mathbb{D}_2$ which may be expressed as:

$$d_1 = \frac{16}{15\sqrt{\pi}} \int_0^\infty \exp\left(-\mathscr{C}_1^2\right) \mathscr{C}_1^3 \left(\frac{5}{2} - \mathscr{C}_1^2 \right) \mathbb{D}_1(\mathscr{C}_1)\, \mathrm{d}\mathscr{C}_1, \tag{169}$$

and:

$$d_{-1} = \frac{16}{15\sqrt{\pi}} \int_0^\infty \exp\left(-\mathscr{C}_2^2\right) \mathscr{C}_2^3 \left(\frac{5}{2} - \mathscr{C}_2^2 \right) \mathbb{D}_2(\mathscr{C}_2)\, \mathrm{d}\mathscr{C}_2. \tag{170}$$

Further, a_1 and a_{-1} are also expressible as integrals on $\tilde{\mathbb{A}}_1$ and $\tilde{\mathbb{A}}_2$ and may be written as:

$$a_1 = \frac{16}{15\sqrt{\pi}} \int_0^\infty \exp\left(-\mathscr{C}_1^2\right) \mathscr{C}_1^3 \left(\frac{5}{2} - \mathscr{C}_1^2 \right) \tilde{\mathbb{A}}_1(\mathscr{C}_1)\, \mathrm{d}\mathscr{C}_1, \tag{171}$$

and:

$$a_{-1} = \frac{16}{15\sqrt{\pi}} \int_0^\infty \exp\left(-\mathscr{C}_2^2\right) \mathscr{C}_2^3 \left(\frac{5}{2} - \mathscr{C}_2^2 \right) \tilde{\mathbb{A}}_2(\mathscr{C}_2)\, \mathrm{d}\mathscr{C}_2. \tag{172}$$

Note that D_{12}, D_T, k_T, λ, $\mathbb{D}_1$, $\mathbb{D}_2$, $\tilde{\mathbb{A}}_1$, $\tilde{\mathbb{A}}_2$, d_0, d_1, d_{-1}, a_1, and a_{-1} are all dependent upon the variable quantities, x_1, x_2, m_1, m_2, σ_1, σ_2, and temperature, T, although these dependencies are not displayed explicitly in the relevant equations. In the m-th order of the expansion, one may write these quantities as:

$$[D_{12}]_m = \frac{1}{2} x_1 x_2 \left(\frac{2kT}{m_0} \right)^{1/2} d_0^{(m)}, \tag{173}$$

$$[D_T]_m = -\frac{5}{4} x_1 x_2 \left(\frac{2kT}{m_0} \right)^{1/2} \left(x_1 M_1^{-1/2} d_1^{(m)} + x_2 M_2^{-1/2} d_{-1}^{(m)} \right), \tag{174}$$

$$[\mathrm{k}_T]_m = \frac{[D_T]_m}{[D_{12}]_m} = -\frac{5}{2} \left(x_1 M_1^{-1/2} d_1^{(m)} + x_2 M_2^{-1/2} d_{-1}^{(m)} \right) / d_0^{(m)}, \tag{175}$$

$$[\lambda]_m = -\frac{5}{4} kn \left(\frac{2kT}{m_0} \right)^{1/2} \left(x_1 M_1^{-1/2} a_1^{(m)} + x_2 M_2^{-1/2} a_{-1}^{(m)} \right), \tag{176}$$

$$\mathbb{D}_1^{(m)}(\mathscr{C}_1) = \sum_{p=0}^{m} d_p^{(m)} S_{3/2}^{(p)}\left(\mathscr{C}_1^2\right) \mathscr{C}_1, \tag{177}$$

$$\mathbb{D}_2^{(m)}(\mathscr{C}_2) = \sum_{p=0}^{m} d_{-p}^{(m)} S_{3/2}^{(p)}\left(\mathscr{C}_2^2\right) \mathscr{C}_2, \tag{178}$$

$$\tilde{\mathbb{A}}_1^{(m)}(\mathscr{C}_1) = \sum_{p=1}^{m} a_p^{(m)} S_{3/2}^{(p)}\left(\mathscr{C}_1^2\right) \mathscr{C}_1, \tag{179}$$

$$\tilde{\mathbb{A}}_2^{(m)}(\mathscr{C}_2) = \sum_{p=1}^{m} a_{-p}^{(m)} S_{3/2}^{(p)}\left(\mathscr{C}_2^2\right) \mathscr{C}_2, \tag{180}$$

$$d_0^{(m)} = \frac{\rho\rho_2}{M_1^{1/2}}\frac{8}{3\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_1^2\right)\mathscr{C}_1^3\mathbb{D}_1^{(m)}(\mathscr{C}_1)\,\mathrm{d}\mathscr{C}_1$$
$$= \frac{\rho\rho_1}{M_2^{1/2}}\frac{8}{3\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_2^2\right)\mathscr{C}_2^3\mathbb{D}_2^{(m)}(\mathscr{C}_2)\,\mathrm{d}\mathscr{C}_2, \tag{181}$$

$$d_1^{(m)} = \frac{16}{15\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_1^2\right)\mathscr{C}_1^3\left(\frac{5}{2}-\mathscr{C}_1^2\right)\mathbb{D}_1^{(m)}(\mathscr{C}_1)\,\mathrm{d}\mathscr{C}_1, \tag{182}$$

$$d_{-1}^{(m)} = \frac{16}{15\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_2^2\right)\mathscr{C}_2^3\left(\frac{5}{2}-\mathscr{C}_2^2\right)\mathbb{D}_2^{(m)}(\mathscr{C}_2)\,\mathrm{d}\mathscr{C}_2, \tag{183}$$

$$a_1^{(m)} = \frac{16}{15\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_1^2\right)\mathscr{C}_1^3\left(\frac{5}{2}-\mathscr{C}_1^2\right)\tilde{\mathbb{A}}_1^{(m)}(\mathscr{C}_1)\,\mathrm{d}\mathscr{C}_1, \tag{184}$$

and:

$$a_{-1}^{(m)} = \frac{16}{15\sqrt{\pi}}\int_0^\infty \exp\left(-\mathscr{C}_2^2\right)\mathscr{C}_2^3\left(\frac{5}{2}-\mathscr{C}_2^2\right)\tilde{\mathbb{A}}_2^{(m)}(\mathscr{C}_2)\,\mathrm{d}\mathscr{C}_2. \tag{185}$$

As a part of this work, we have carried out two basic sets of calculations:

(i) First, we have conducted a comparison of our results from the current work with the results previously reported by Takata et al. [19]. To conduct this comparison, we have adapted our results so as to present them using the same non-dimensionalization scheme employed by Takata et al. and have then plotted our adapted results in graphical form with the same scaling and for the same set of virtual gas mixtures reported by Takata et al.
(ii) Second, using a non-dimensionalization/normalization scheme similar to that of Chapman and Cowling [1] which normalizes the transport coefficients relative to their first-order approximate results, we have obtained a comprehensive set of order 70 results for all binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe. Additionally, we present extrapolated values of the transport coefficients that have been obtained by applying the *Mathematica*® function `SequenceLimit` to the sequence of normalized transport coefficient results corresponding to orders 1 through 70.

In the work of Takata et al. [19], the authors have considered cases involving a selection of 'virtual' gas mixtures for which the size and mass ratios of the constituents are general values only and do not reflect the sizes and masses of specific gas constituents. The size ratios that have been considered in this work include $\sigma_2/\sigma_1 = \frac{1}{2}, 1, 2$ and the mass ratios that have been considered include $m_2/m_1 = 1, 2, 3, 4, 5, 8, 10$. Mole fractions are specified by $x_1 \in (0, 1)$ with $x_1 + x_2 = 1$. The authors define non-dimensional diffusion and thermal diffusion coefficients in a manner that may be expressed in the following way:

$$\hat{D}_{12} = f_3 D_{12}, \tag{186}$$
$$\hat{D}_T = f_3 D_T, \tag{187}$$

where:

$$f_3 = 2\sqrt{2\pi}\sqrt{\frac{m_1}{2kT}}\,n\sigma_1^2, \tag{188}$$

and a non-dimensional thermal conductivity coefficient as:

$$\hat{\lambda} = \frac{f_3}{\sqrt{2}kn}\lambda. \tag{189}$$

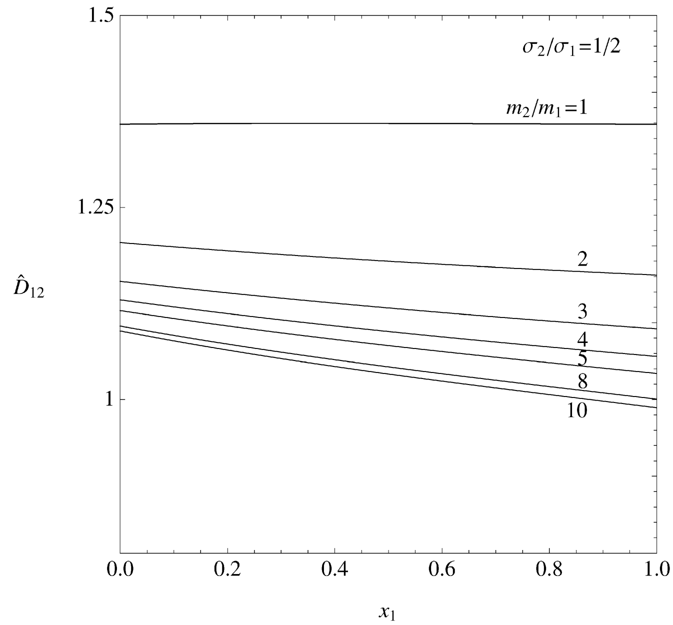

Fig. 1. The $\hat{D}_{12}$ functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 1/2$.

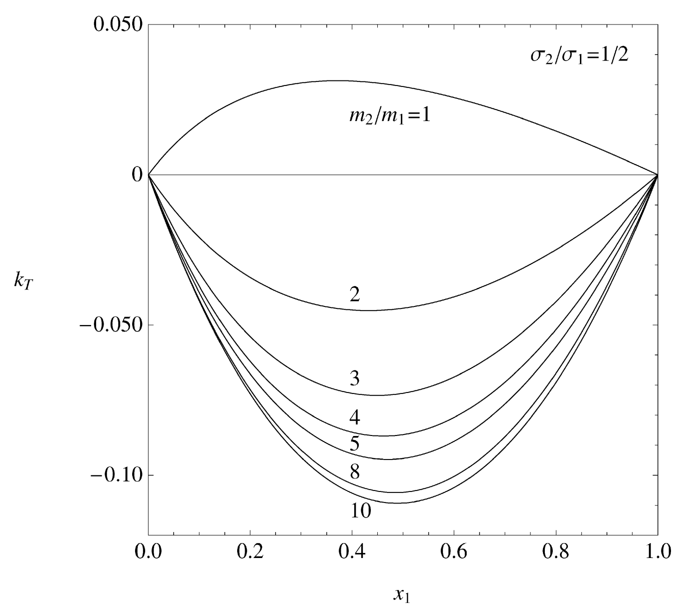

Fig. 2. The k_T functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 1/2$.

Thus, $\hat{D}_{12}$, $\hat{D}_T$, and $\hat{\lambda}$ depend only upon the masses, m_1 and m_2, the molecular diameters, σ_1 and σ_2, and the number densities, n_1 and n_2. Dependence upon temperature, T, has been totally eliminated for rigid-sphere molecules which is quite convenient because the rigid-sphere model is known to exhibit an unrealistic temperature dependence. The authors have presented their results graphically and have not given their precise numerical results explicitly. We have reported here our results, in the same format and for exactly the same cases considered by Takata et al., in Figs. 1–9 where our reported results conform to the same non-dimensionalization scheme described in Eqs. (186)–(189). Note that instead of presenting $\hat{D}_T$ directly, Takata et al. present results for k_T which yields $\hat{D}_T$ from the $\hat{D}_{12}$ results via $\hat{D}_T = k_T\hat{D}_{12}$ and that we have presented our comparison results accordingly. Our graphical results are indis-

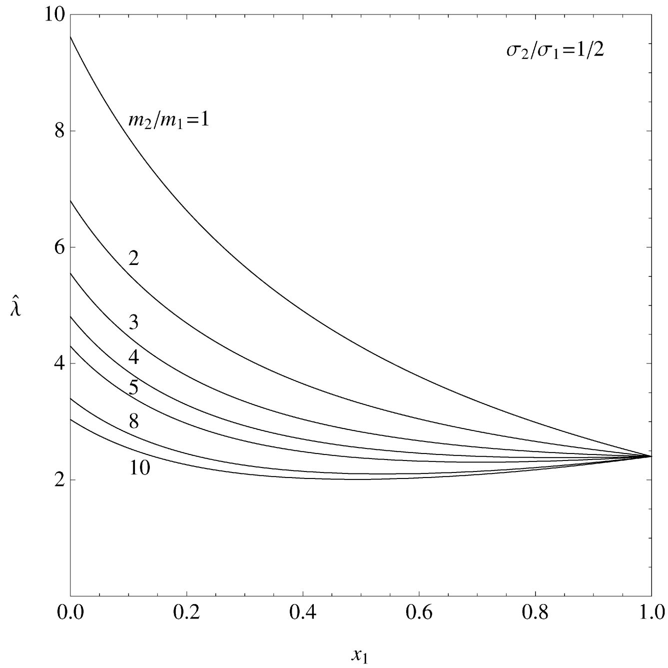

Fig. 3. The $\hat{\lambda}$ functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 1/2$.

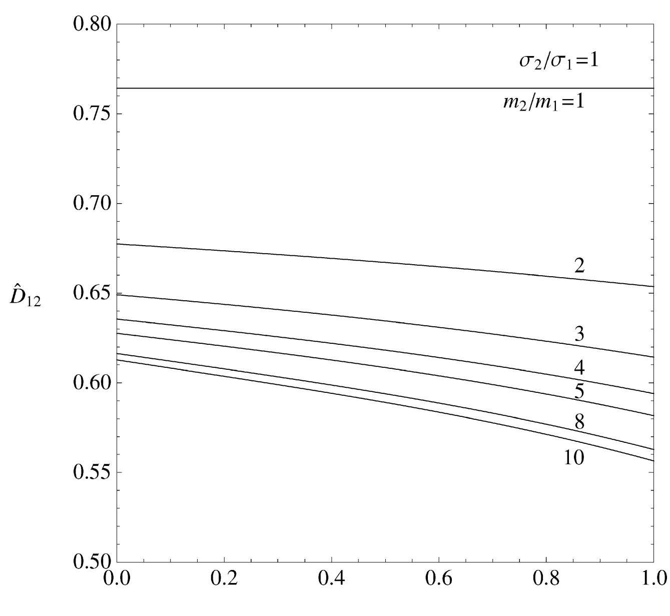

Fig. 4. The $\hat{D}_{12}$ functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 1$.

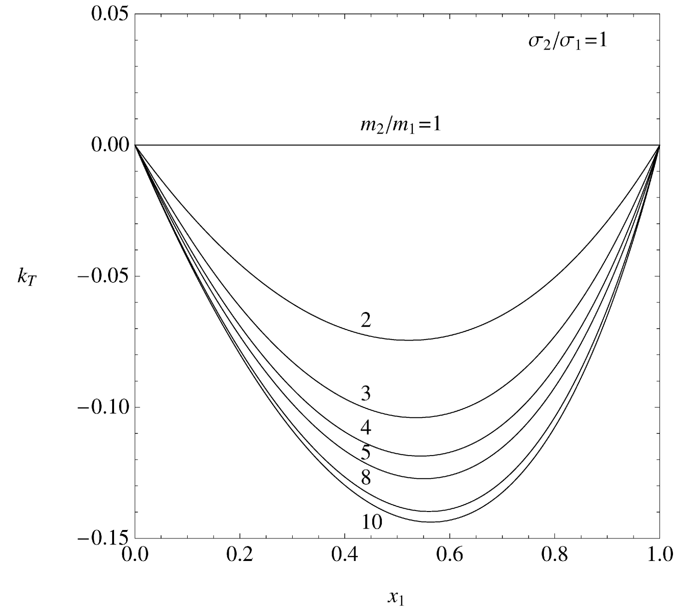

Fig. 5. The k_T functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 1$.

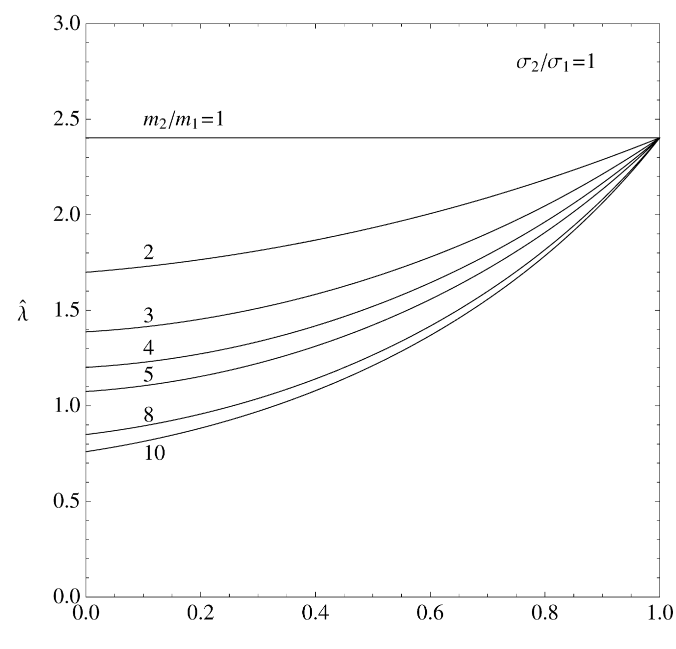

Fig. 6. The $\hat{\lambda}$ functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 1$.

tinguishable from those of Takata et al., and thus, we assume that they have obtained precise results for the cases they considered.

Lastly, we present the primary results of this work for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe. We note that a more concise way to report transport coefficient results may be achieved by normalizing them in the following manner:

$$[D_{12}]_m = [D_{12}]_1 [D_{12}]_m^\star, \tag{190}$$

$$[D_T]_m = [D_T]_1 [D_T]_m^\star, \tag{191}$$

$$[\lambda]_m = [\lambda]_1 [\lambda]_m^\star, \tag{192}$$

where:

$$[D_{12}]_m^\star \equiv \frac{[D_{12}]_m}{[D_{12}]_1}, \tag{193}$$

$$[D_T]_m^\star \equiv \frac{[D_T]_m}{[D_T]_1}, \tag{194}$$

$$[\lambda]_m^\star \equiv \frac{[\lambda]_m}{[\lambda]_1}, \tag{195}$$

and where $[D_{12}]_1$, $[D_T]_1$, $[\lambda]_1$ are the transport coefficients of the mixture computed with first-order approximations ($m = 1$). In the general case, the first-order transport coefficients can be explicitly expressed as:

$$[D_{12}]_1 = \frac{1}{2} x_1 x_2 \left(\frac{2kT}{m_0}\right)^{1/2} d_0^{(1)}, \tag{196}$$

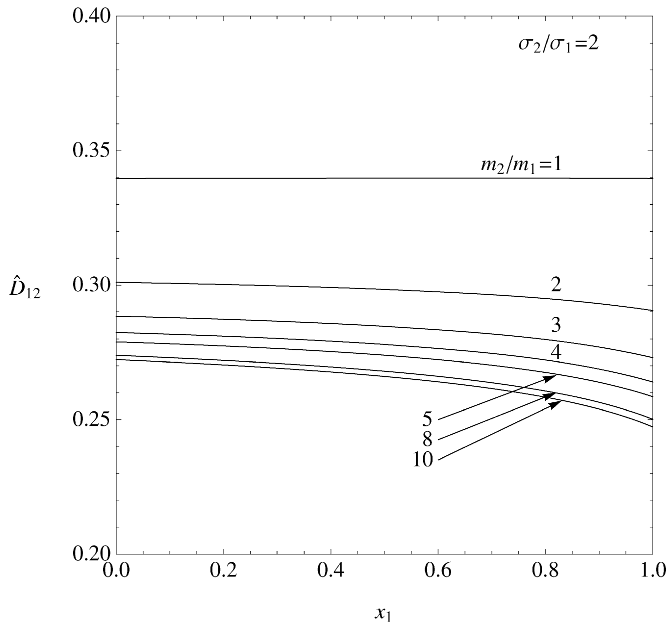

Fig. 7. The $\hat{D}_{12}$ functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 2$.

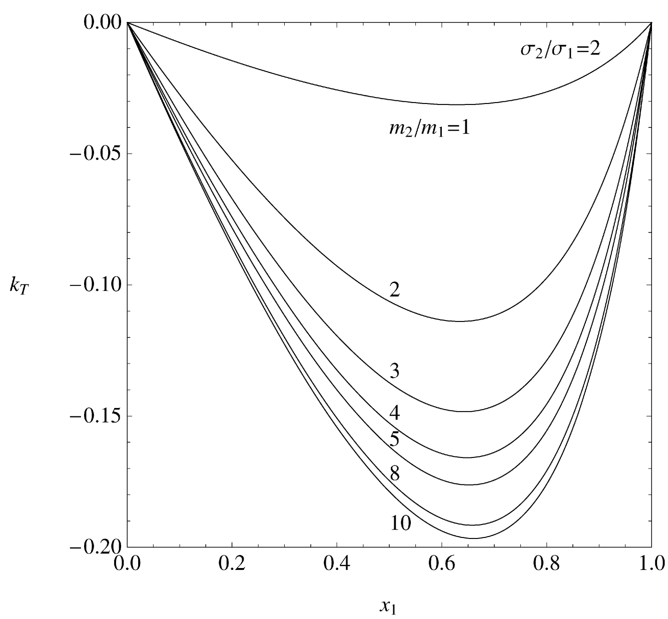

Fig. 8. The k_T functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 2$.

$$[D_T]_1 = -\frac{5}{4}x_1x_2\left(\frac{2kT}{m_0}\right)^{1/2}\left(x_1M_1^{-1/2}d_1^{(1)} + x_2M_2^{-1/2}d_{-1}^{(1)}\right), \tag{197}$$

$$[\lambda]_1 = -\frac{5}{4}kn\left(\frac{2kT}{m_0}\right)^{1/2}\left(x_1M_1^{-1/2}a_1^{(1)} + x_2M_2^{-1/2}a_{-1}^{(1)}\right), \tag{198}$$

where:

$$d_0^{(1)} = \delta_0(a_{-1-1}a_{11} - a_{-11}a_{1-1})/\Delta_D, \tag{199}$$

$$d_1^{(1)} = -\delta_0(a_{-1-1}a_{10} - a_{-10}a_{1-1})/\Delta_D, \tag{200}$$

$$d_{-1}^{(1)} = -\delta_0(a_{-10}a_{11} - a_{-11}a_{10})/\Delta_D, \tag{201}$$

$$a_1^{(1)} = (a_{-1-1}\alpha_1 - \alpha_{-1}a_{1-1})/\Delta_\lambda, \tag{202}$$

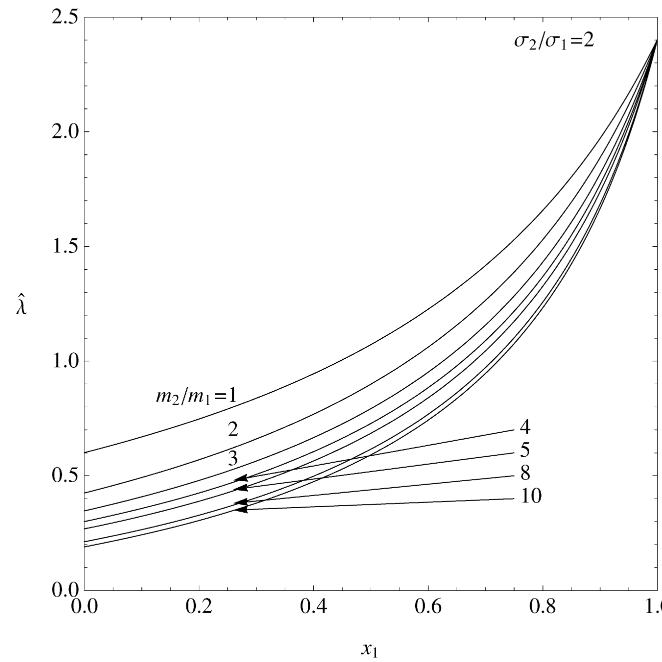

Fig. 9. The $\hat{\lambda}$ functions calculated in this work for comparison with the values reported by Takata et al. [19] for $\sigma_2/\sigma_1 = 2$.

and:

$$a_{-1}^{(1)} = (\alpha_{-1}a_{11} - a_{-11}\alpha_1)/\Delta_\lambda, \tag{203}$$

in which the determinants, Δ_D and Δ_λ, can be expressed as:

$$\Delta_D = a_{-1-1}a_{00}a_{11} + 2a_{-10}a_{01}a_{-11} - a_{-11}^2a_{00} - a_{-10}^2a_{11} - a_{01}^2a_{-1-1}, \tag{204}$$

and:

$$\Delta_\lambda = a_{-1-1}a_{11} - a_{-11}^2, \tag{205}$$

after the appropriate symmetries have been employed, where completely general explicit expressions for the necessary a_{pq} values have already been given in terms of the omega integrals in Eqs. (112)–(120), and where the quantities δ_0, α_1, α_{-1} have been given in Eqs. (62) and (65). Additionally, one may also normalize the quantities d_0^m, d_{-1}^m, d_1^m, a_{-1}^m, and a_1^m in the following manner:

$$d_0^{(m)\star} = \frac{d_0^{(m)}}{d_0^{(1)}}, \qquad d_{-1}^{(m)\star} = \frac{d_{-1}^{(m)}}{d_0^{(1)}}, \qquad d_1^{(m)\star} = \frac{d_1^{(m)}}{d_0^{(1)}},$$
$$a_{-1}^{(m)\star} = \frac{a_{-1}^{(m)}}{a_{-1}^{(1)}}, \qquad a_1^{(m)\star} = \frac{a_1^{(m)}}{a_1^{(1)}}, \tag{206}$$

such that the transport coefficients may be expressed as:

$$[D_{12}]_m = [D_{12}]_m^\star[D_{12}]_1 = \frac{1}{2}x_1x_2\left(\frac{2kT}{m_0}\right)^{1/2}d_0^{(m)\star}d_0^{(1)}, \tag{207}$$

$$[D_{12}]_m^\star \equiv \frac{[D_{12}]_m}{[D_{12}]_1} = \frac{d_0^{(m)\star}d_0^{(1)}}{d_0^{(1)\star}d_0^{(1)}} = d_0^{(m)\star}, \tag{208}$$

$$[D_T]_m = [D_T]_m^\star[D_T]_1 = -\frac{5}{4}x_1x_2\left(\frac{2kT}{m_0}\right)^{1/2} \times \left(x_1M_1^{-1/2}d_1^{(m)\star}d_0^{(1)} + x_2M_2^{-1/2}d_{-1}^{(m)\star}d_0^{(1)}\right), \tag{209}$$

Table 1
Order 1 values of the thermal conductivity coefficients, $[\lambda]_1$, for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe as functions of the mixture fractions of the lighter constituents.

Mixture	x_1	$[\lambda]_1 \times 10^{-2}$ (erg cm^{-1} s^{-1} K^{-1})	$a_1^{(1)} \times 10^6$ (cm)	$a_{-1}^{(1)} \times 10^6$ (cm)
He:Ne	10^{-100}	45.230993622931949432092	−15.235762082277183907967	−20.560567836453608501773
	10^{-12}	45.230993622959561499966	−15.235762082282910757147	−20.560567836452510851540
	10^{-9}	45.230993650544017344413	−15.235762088004033091935	−20.560567835355958272230
	10^{-6}	45.231021235038011481513	−15.235767809130253987916	−20.560566738806778066970
	10^{-3}	45.258643891588442209066	−15.241492823283853661098	−20.559473589249784694647
	0.1	48.387938101542520763805	−15.849285472703085686244	−20.485419586314615491397
	0.2	52.400524509330325834942	−16.553245389870214843796	−20.482507692957853305955
	0.3	57.387988310715000729415	−17.363463981769427446356	−20.557967623931769897757
	0.4	63.507872139255285497994	−18.300295079950876996768	−20.720579390524086569672
	0.5	70.969261682002938249290	−19.390261245686244642328	−20.982594289396504111620
	0.6	80.052478276483582685685	−20.668475917802252031174	−21.360981402079155359071
	0.7	91.139095345248514789382	−22.182323557218511221395	−21.879341994421661891460
	0.8	104.75907270125660531323	−23.997227989269715198034	−22.570928518511133383181
	0.9	121.66747324170925742034	−26.206031530731653628869	−23.483570041227100741296
	$1-10^{-3}$	142.73506214590358812824	−28.914309063342466166423	−24.674248929098008473390
	$1-10^{-6}$	142.97459300150129227556	−28.944894163038547987782	−24.688039283814368562201
	$1-10^{-9}$	142.97483283255399563591	−28.944924784469089457678	−24.688053093894623570332
	$1-10^{-12}$	142.97483307238534888032	−28.944924815090556372077	−24.688053107704723572918
	$1-10^{-100}$	142.97483307262542030529	−28.944924815121208491147	−24.688053107718547496864
He:Ar	10^{-100}	16.256824822751970282052	−8.4323761933253591554156	−10.397402827308215058740
	10^{-12}	16.256824822782258518204	−8.4323761933310914217505	−10.397402827311344442496
	10^{-9}	16.256824853040206460143	−8.4323761990576254944640	−10.397402830437598816961
	10^{-6}	16.256855111014219845242	−8.4323819255957932514409	−10.397405956693682862583
	10^{-3}	16.287139175867276683665	−8.4381125617645973432374	−10.400533923526317980475
	0.1	19.567541902225391688039	−9.0497831348698777801409	−10.728712359301581664450
	0.2	23.539928793551250511756	−9.7704620780345899163411	−11.102639591873315825266
	0.3	28.361648838239252336184	−10.622974627493155043775	−11.530417466566811180428
	0.4	34.296989480121495689575	−11.647502773190528988051	−12.027816742107354833584
	0.5	41.730273698976712103014	−12.902429339929236049241	−12.617713277298762831249
	0.6	51.242542064205417756578	−14.475961424479338553857	−13.334622552473305155498
	0.7	63.755608338092135001721	−16.507965933497788504917	−14.233217844375882064982
	0.8	80.824394494466303843452	−19.234280433993090239969	−15.405615796807564779150
	0.9	105.29247888187096812075	−23.086077638689591412750	−17.020137823383066122898
	$1-10^{-3}$	142.49863276013777918019	−28.871252488140866362500	−19.390899252221085359806
	$1-10^{-6}$	142.97435560731035884862	−28.944850950838639729143	−19.420776386431113933909
	$1-10^{-9}$	142.97483259515883693443	−28.944924741256733465538	−19.420806338612713724796
	$1-10^{-12}$	142.97483307214795372065	−28.944924815047344015929	−19.420806368564970567536
	$1-10^{-100}$	142.97483307262542030529	−28.944924815121208491147	−19.420806368594952806693
He:Kr	10^{-100}	8.5231373968991121279109	−6.4236103707880117230623	−7.8951055393209531462766
	10^{-12}	8.5231373969264292591221	−6.4236103707929621870928	−7.8951055393247607986081
	10^{-9}	8.5231374242162433616969	−6.4236103757384757574486	−7.8951055431286054798546
	10^{-6}	8.5231647140528833641500	−6.4236153212558858827084	−7.8951093469753387544248
	10^{-3}	8.5504771062300904651042	−6.4285646814303818757128	−7.8989152469822635416820
	0.1	11.500121513991510257611	−6.9603421585728587656743	−8.2977474097886881544248
	0.2	15.061120659414146582782	−7.5958476688587996238944	−8.7503260274415129260526
	0.3	19.389887129167812189482	−8.3603033300904314097599	−9.2645372955727007805849
	0.4	24.754821876384359333707	−9.2976057880465450121802	−9.8566582674279931013899
	0.5	31.563620466317447480319	−10.474180161884007673980	−10.550286267971477423592
	0.6	40.466352048246442605190	−11.995577994852039035705	−11.381429471936427427775
	0.7	52.565641753550265656206	−14.040293586592805681931	−12.408763866052933776804
	0.8	69.890789514293486821422	−16.935998577652846538887	−13.736562819065077602709
	0.9	96.624397468245058785710	−21.356690336198821618550	−15.573647048581970959196
	$1-10^{-3}$	142.34477685738630638930	−28.842196175955174114828	−18.381955737403055609076
	$1-10^{-6}$	142.97420073031560194734	−28.944821718258313120105	−18.418719118174360335215
	$1-10^{-9}$	142.97483244028081616119	−28.944924712023976049107	−18.418755994957361669721
	$1-10^{-12}$	142.97483307199307569885	−28.944924815018111258336	−18.418756031834258468844
	$1-10^{-100}$	142.97483307262542030529	−28.944924815121208491147	−18.418756031871172279568
He:Xe	10^{-100}	4.9216330167821820114007	−4.9995786142542142848018	−5.7065238518632256560958
	10^{-12}	4.9216330168049404976975	−4.9995786142583368968532	−5.7065238518666860743622
	10^{-9}	4.9216330395406683274725	−4.9995786183768263396286	−5.7065238553236439246953
	10^{-6}	4.9216557752877341925181	−4.9995827368696712091289	−5.7065273122836672308411
	10^{-3}	4.9444107746153191591834	−5.0037046346245612308914	−5.7099864467041746748616
	0.1	7.4076847095035792085458	−5.4489654105839980495054	−6.0758466337510474738419
	0.2	10.398903341303874843029	−5.9873226115769091923634	−6.4988297001281620310023
	0.3	14.064907578058101517237	−6.6440247940744701845761	−6.9889418372961812327820
	0.4	18.660212299585055691285	−7.4629944596769687719136	−7.5649850437556362081617
	0.5	24.584645450650940978795	−8.5129895533409595520534	−8.2542918083046264051475
	0.6	32.503729602725701164060	−9.9080644914865578026027	−9.0987581436560020617886
	0.7	43.612056052181092253450	−11.852263720875267835240	−10.167389509661436914233
	0.8	60.286255627633844887813	−14.750278683598265835360	−11.586040751847823852759
	0.9	88.018408944825273916226	−19.534749737203017957673	−13.622308244103636492321

(continued on next page)

Table 1 (*continued*)

Mixture	x_1	$[\lambda]_1 \times 10^{-2}$ $(\mathrm{erg\,cm^{-1}\,s^{-1}\,K^{-1}})$	$a_1^{(1)} \times 10^6$ (cm)	$a_{-1}^{(1)} \times 10^6$ (cm)
	$1-10^{-3}$	142.16078786736296942069	−28.80596750806466967410 5	−16.96254478571167964510 8
	$1-10^{-6}$	142.97401506286978174515	−28.94478518595438998808 9	−17.00872609133057039660 1
	$1-10^{-9}$	142.97483225461168087819	−28.94492467549136655849 4	−17.00877245319957764087 1
	$1-10^{-12}$	142.97483307180740656188	−28.94492481498157864854 0	−17.00877249956162803443 0
	$1-10^{-100}$	142.97483307262542030529	−28.94492481512120849114 7	−17.00877249960803649346 4
Ne:Ar	10^{-100}	16.256824822751970282052	−11.278569674291800245673	−10.397402827308215058740
	10^{-12}	16.256824822765709502436	−11.278569674296427070979	−10.397402827311530872247
	10^{-9}	16.256824836491190673602	−11.278569678918625553815	−10.397402830624028568129
	10^{-6}	16.256838561980024210522	−11.278574301119442891044	−10.397406143123434209442
	10^{-3}	16.270571716978149590602	−11.283198837600027865677	−10.400720353139180159597
	0.1	17.711441480628205101615	−11.765844845579760833382	−10.746994314255090697843
	0.2	19.345192957126212032129	−12.307744311033637556583	−11.136584180121064814218
	0.3	21.189377090608154056439	−12.913860377980460475746	−11.573188337447615657630
	0.4	23.283083878352463674041	−13.596172877788376751166	−12.065569065724198878882
	0.5	25.675780525893720612200	−14.369841238085014142912	−12.624814374960686809407
	0.6	28.430998757976950359101	−15.254334293300828485683	−13.265164408995026769650
	0.7	31.631714676888278022561	−16.275078699973845245203	−14.005217183845738207578
	0.8	35.388402651809414235266	−17.465926688192154281350	−14.869733631996836354835
	0.9	39.851453929045159259045	−18.872961184789427137348	−15.892420871354822097155
	$1-10^{-3}$	45.171767024784271845005	−20.542028825568351516667	−17.106877514578339270820
	$1-10^{-6}$	45.230934336090988009304	−20.560549278996124803739	−17.120360095516103671634
	$1-10^{-9}$	45.230993563645048168086	−20.560567817896132553163	−17.120373591591834993868
	$1-10^{-12}$	45.230993622872662530768	−20.560567836435051025806	−17.120373605087924233380
	$1-10^{-100}$	45.230993622931949432092	−20.560567836453608501773	−17.120373605101433832232
Ne:Kr	10^{-100}	8.5231373968991121279109	−7.6363531059823579248179	−7.8951055393209531462766
	10^{-12}	8.5231373969097498650592	−7.6363531059866818450029	−7.8951055393231408560234
	10^{-9}	8.5231374075368492850514	−7.6363531103062781126590	−7.8951055415086628947279
	10^{-6}	8.5231480346451129255224	−7.6363574299054409249450	−7.8951077270322766918259
	10^{-3}	8.5337839923515389852404	−7.6406799261444439733855	−7.8972948267788117457903
	0.1	9.6816446629329232159696	−8.0997827393270943273310	−8.1307575358357003282027
	0.2	11.058253391100742656625	−8.6347741283055621065517	−8.4053106528001039118828
	0.3	12.706589402535584522006	−9.2590913255519105909786	−8.7283885085970150107962
	0.4	14.699649191321615002990	−9.9969120798985402418962	−9.1130899049884847827577
	0.5	17.139409031809005775083	−10.882009511840432735664	−9.5777127663331974534090
	0.6	20.172784347253153234791	−11.963036424830443512359	−10.148617358255373686608
	0.7	24.019441412358268379554	−13.312734044410849797995	−10.865215894489722195435
	0.8	29.023181278572522108607	−15.044945759179923940236	−11.789191158569017289659
	0.9	35.754038985083526487585	−17.348423206875718301086	−13.022813787899885084459
	$1-10^{-3}$	45.117319267409175033020	−20.522189730117384920954	−14.728246477554520712732
	$1-10^{-6}$	45.230879720269855532863	−20.560529382781939616877	−14.748877261394815254149
	$1-10^{-9}$	45.230993509029058582308	−20.560567797999861118782	−14.748897933128143494481
	$1-10^{-12}$	45.230993622818046541014	−20.560567836415154754315	−14.748897953799917852820
	$1-10^{-100}$	45.230993622931949432092	−20.560567836453608501773	−14.748897953820610319686
Ne:Xe	10^{-100}	4.9216330167821820114007	−5.4720824083065644222746	−5.7065238518632256560958
	10^{-12}	4.9216330167910753344736	−5.4720824083103660634244	−5.7065238518652860191550
	10^{-9}	4.9216330256755050920458	−5.4720824121082055749368	−5.7065238539235887165712
	10^{-6}	4.9216419101129537108420	−5.4720862099505327745296	−5.7065259122275747948506
	10^{-3}	4.9305340445041817731054	−5.4758868700761456446901	−5.7085855058141620135230
	0.1	5.8943304123945284371890	−5.8827146795569607295900	−5.9264742056499951673442
	0.2	7.0637884852976528622716	−6.3650285191824847675999	−6.1790243293336718962709
	0.3	8.4881803600790372362734	−6.9398256012346274158675	−6.4734036672564679278856
	0.4	10.250696016066015082279	−7.6368325548298040000293	−6.8227891562343736252111
	0.5	12.474578210118997342807	−8.5000637322958383497389	−7.2466763175902791967620
	0.6	15.350481503466251519404	−9.5975790619347496365833	−7.7751935180136651718561
	0.7	19.189801214506980256999	−11.040504167154340729277	−8.4575112858294888041802
	0.8	24.538136700576852048404	−13.023497591753890053528	−9.3797338636491877774057
	0.9	32.446305757994103126766	−15.921409127954569084559	−10.707634067865405585515
	$1-10^{-3}$	45.064572534188571392425	−20.500408371444762677280	−12.779512828355054881396
	$1-10^{-6}$	45.230826695312209880577	−20.560507496227806083556	−12.806568545877326680384
	$1-10^{-9}$	45.230993456003821635549	−20.560567776113201394297	−12.806595681576273308540
	$1-10^{-12}$	45.230993622765021303788	−20.560567836393268094485	−12.806595708712052477232
	$1-10^{-100}$	45.230993622931949432092	−20.560567836453608501773	−12.806595708739215419423
Ar:Kr	10^{-100}	8.5231373968991121279109	−7.4818474741237029603714	−7.8951055393209531462766
	10^{-12}	8.5231373969029341875270	−7.4818474741252781462408	−7.8951055393215524524053
	10^{-9}	8.5231374007211717466770	−7.4818474756988888306684	−7.8951055399202592757971
	10^{-6}	8.5231412189614106637511	−7.4818490493104672297140	−7.8951061386278845657237
	10^{-3}	8.5269621396743140484873	−7.4834235551235611362844	−7.8957056483130435568287
	0.1	8.9329102909859355939048	−7.6485902786934903311249	−7.9632347636237217268826
	0.2	9.4010877599567242765786	−7.8349878353158002011363	−8.0485309780164206407939
	0.3	9.9333299875374637601988	−8.0431114258673535595901	−8.1523559374172696018765
	0.4	10.536464898710989270822	−8.2754394056072791193732	−8.2763890654171941217355
	0.5	11.218740263410843013411	−8.5349470601747909635337	−8.4226924554273676363702

Table 1 (*continued*)

Mixture	x_1	$[\lambda]_1 \times 10^{-2}$ $(\mathrm{erg\,cm^{-1}\,s^{-1}\,K^{-1}})$	$a_1^{(1)} \times 10^6$ (cm)	$a_{-1}^{(1)} \times 10^6$ (cm)
	0.6	11.99015184475924218738 1	−8.82522328394754358830 82	−8.59379599226403629787 07
	0.7	12.862874418724216478170	−9.1506235836326133702135	−8.7928096162737745099922
	0.8	13.851834117799969218978	−9.5164730222820670046879	−9.0235728084330363076675
	0.9	14.975478121543663001491	−9.9293389338326755124480	−9.2908559914623257257164
	$1-10^{-3}$	16.243141248290820560549	−10.392417184214444186475	−9.5973042214073287074964
	$1-10^{-6}$	16.256811129887982475329	−10.397397838406575321548	−9.6006323280668454416518
	$1-10^{-9}$	16.256824809059096999314	−10.397402822319310158539	−9.6006356586639083546186
	$1-10^{-12}$	16.256824822738277408760	−10.397402827303226153836	−9.6006356619945079093442
	$1-10^{-100}$	16.256824822751970282052	−10.397402827308215058740	−9.6006356619978418428349
Ar:Xe	10^{-100}	4.9216330167821820114007	−5.2619169547344416197986	−5.7065238518632256560958
	10^{-12}	4.9216330167862284280765	−5.2619169547366104652538	−5.7065238518640845935197
	10^{-9}	4.9216330208285986906293	−5.2619169569032870763598	−5.7065238527221630809237
	10^{-6}	4.9216370632022921194114	−5.2619191235812403454713	−5.7065247108015586265068
	10^{-3}	4.9256828694524455140116	−5.2640871444160887618459	−5.7073836987615050540377
	0.1	5.3624023234483396520012	−5.4929980379252181377905	−5.8019575206202995458849
	0.2	5.8834912127967103951688	−5.7559022797220877600666	−5.9185059504163186516992
	0.3	6.4994915589469157390005	−6.0568113199472182397758	−6.0598601974930554365307
	0.4	7.2292629515447565008896	−6.4036818694714971740300	−6.2308027562305588306977
	0.5	8.0975603471613252400570	−6.8069256597050759336150	−6.4376227476122423480447
	0.6	9.1374745762242457607419	−7.2804286351167788473440	−6.6887385726415273079721
	0.7	10.394201411324449634717	−7.8431246584790138290339	−6.9956596392619818189366
	0.8	11.931066234818637846176	−8.5215106938464042986770	−7.3745237457600012775752
	0.9	13.839552617694560252194	−9.3538332436019900995030	−7.8486562973754484222293
	$1-10^{-3}$	16.229515491701221314498	−10.385663270953160019587	−8.4461957301908199751629
	$1-10^{-6}$	16.256797477853908590549	−10.397391072959357326807	−8.4530351690218901168399
	$1-10^{-9}$	16.256824795407036609312	−10.397402815553851390114	−8.4530420175570447495306
	$1-10^{-12}$	16.256824822724625348344	−10.397402827296460695056	−8.4530420244055890117226
	$1-10^{-100}$	16.256824822751970282052	−10.397402827308215058740	−8.4530420244124444113935
Kr:Xe	10^{-100}	4.9216330167821820114007	−5.8742656972102459407328	−5.7065238518632256560958
	10^{-12}	4.9216330167844884116317	−5.8742656972115813664334	−5.7065238518642535185051
	10^{-9}	4.9216330190885822433317	−5.8742656985456716417796	−5.7065238528910880657978
	10^{-6}	4.9216353231833094225720	−5.8742670326364177188001	−5.7065248797260305866633
	10^{-3}	4.9239403136603149537849	−5.8756015942589678065124	−5.7075521100164137746613
	0.1	5.1615166149119851359587	−6.0126708821911448642501	−5.8133901134098492922604
	0.2	5.4210836674008305545319	−6.1614448705610871210206	−5.9289456012444536681961
	0.3	5.7023202211719088194515	−6.3216554985825581568541	−6.0540687951402705482832
	0.4	6.0075048950809476333028	−6.4945271177771954731887	−6.1897675231965742449094
	0.5	6.3392640549323421272936	−6.6814701455741088264609	−6.3372033578847108401428
	0.6	6.7006400046359049002837	−6.8841175853785941048957	−6.4977217671356540152647
	0.7	7.0951759199421548309279	−7.1043705114678023232609	−6.6728896681092584688022
	0.8	7.5270225226304726293178	−7.3444551949324545797838	−6.8645425934489220393628
	0.9	8.0010732742195190087037	−7.6069955006558297700437	−7.0748444673779140409638
	$1-10^{-3}$	8.5176573495455740352488	−7.8920862166319807651401	−7.3039342695132394396857
	$1-10^{-6}$	8.5231319141135727158808	−7.8951025185388736861725	−7.3063616759756412738790
	$1-10^{-9}$	8.5231373914163238490512	−7.8951055363001696067498	−7.3063641045922375344366
	$1-10^{-12}$	8.5231373968936293396293	−7.8951055393179323627356	−7.3063641070208553413890
	$1-10^{-100}$	8.5231373968991121279109	−7.8951055393209531462766	−7.3063641070232863902460

Table 2
Normalized order 70 values of the thermal conductivity coefficients, $[\lambda]_{70}^{\star}$, for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe as functions of the mixture fractions of the lighter constituents.

Mixture	x_1	$[\lambda]_{70}^{\star}$	$a_1^{(70)\star}$	$a_{-1}^{(70)\star}$
He:Ne	10^{-100}	1.0252181683234523152732	1.0872730381181951998864	1.0252181683234523152732
	10^{-12}	1.0252181683234992998780	1.0872730381181435134042	1.0252181683233960496321
	10^{-9}	1.0252181683704369201471	1.0872730380665087176882	1.0252181682671866741921
	10^{-6}	1.0252182153080706373937	1.0872729864317084099620	1.0252181120578412679165
	10^{-3}	1.0252651662653715155855	1.0872213470300225895264	1.0251619327268161039715
	0.1	1.0299336122181635631348	1.0820491736037352522999	1.0198741240580900360365
	0.2	1.0342711214903130897172	1.0766814477738833850159	1.0150314435617134428974
	0.3	1.0377441997211695641340	1.0711266832558147250976	1.0106120503297540613266
	0.4	1.0400309786138353014717	1.0653496646035295659507	1.0065581626514015571345
	0.5	1.0409508018112383446086	1.0593224379890634405206	1.0028303128966243620649
	0.6	1.0404352117350897703442	1.0530248668697698873245	0.9994073868850399914806 0
	0.7	1.0385000964002596553389	1.0464468636241766023130	0.9962890415854609009221 8
	0.8	1.0352239452661135695676	1.0395932460769784528097	0.9935015015283871207192 3
	0.9	1.0307359485698434698333	1.0324931689235703671378	0.9911088730910625798972 4
	$1-10^{-3}$	1.0252776790844339602297	1.0252913852507034406367	0.9892499813929333280018 2
	$1-10^{-6}$	1.0252182278733048580938	1.0252182415422762545843	0.9892344369014389454441 0
	$1-10^{-9}$	1.0252181683830022068565	1.0252181683966711410760	0.9892344213932212332217 8
	$1-10^{-12}$	1.0252181683235118651648	1.0252181683235255340990	0.9892344213777130518309 6
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	0.9892344213776975281259 0

(*continued on next page*)

Table 2 (*continued*)

Mixture	x_1	$[\lambda]^{\star}_{70}$	$a_1^{(70)\star}$	$a_{-1}^{(70)\star}$
He:Ar	10^{-100}	1.0252181683234523152732	1.1246786406007335143153	1.0252181683234523152732
	10^{-12}	1.0252181683236561715647	1.1246786406006701074103	1.0252181683234013407534
	10^{-9}	1.0252181685273086064873	1.1246786405373266092694	1.0252181682724777954824
	10^{-6}	1.0252183721794181535275	1.1246785771938095349215	1.0252181173489402139158
	10^{-3}	1.0254216991572284250253	1.1246152147323287631518	1.0251672014989919891018
	0.1	1.0425705712840335307652	1.1181370582753568669298	1.0201960573132133428433
	0.2	1.0546112190247962792290	1.1111443026133668172420	1.0153188819417827259622
	0.3	1.0622186934333221356379	1.1036160984818831341805	1.0105801762743270389436
	0.4	1.0660150643963094597939	1.0954518501239807139867	1.0059777213665793032351
	0.5	1.0664353047249363579152	1.0865308760199950868443	1.0015169493799491926096
	0.6	1.0637717732775435996482	1.0767088593453272501498	0.99721829880488748739083
	0.7	1.0582057049773514561943	1.0658168822635821228506	0.99313351732720117916212
	0.8	1.0498390730900356424888	1.0536708506370629141863	0.98938389885166144994244
	0.9	1.0387565130004953058065	1.0401177709862649927974	0.98625807935557245604182
	$1-10^{-3}$	1.0253633916545607905637	1.0253720873581989609769	0.98450190065385425281460
	$1-10^{-6}$	1.0252183136233177377633	1.0252183222720458698199	0.98449596124798519707206
	$1-10^{-9}$	1.0252181684687522569083	1.0252181684774009380899	0.98449595546752178497056
	$1-10^{-12}$	1.0252181683235976152149	1.0252181683236062638960	0.98449595546174148104042
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	0.98449595546173569495056
He:Kr	10^{-100}	1.0252181683234523152732	1.1503483128349767016658	1.0252181683234523152732
	10^{-12}	1.0252181683238784179016	1.1503483128349034898519	1.0252181683234125864371
	10^{-9}	1.0252181687495549424323	1.1503483127617648877133	1.0252181682837234791772
	10^{-6}	1.0252185944248187223073	1.1503482396231392264108	1.0252181285946128701169
	10^{-3}	1.0256430118362083088582	1.1502750774596285027042	1.0251784361346283416015
	0.1	1.0575546356723539162786	1.1427758039023226716517	1.0212116188191119600021
	0.2	1.0756772661006479631344	1.1346308375906564585308	1.0171379339144574419631
	0.3	1.0850658945914598437344	1.1257912716051758577229	1.0129995901852534725038
	0.4	1.0884310310918071922230	1.1161048123478322696479	1.0088033082426836811597
	0.5	1.0871886867398702683191	1.1053778661295186316289	1.0045631060246112958264
	0.6	1.0820591608580884631148	1.0933607214406165760273	1.0003069198342868772880
	0.7	1.0733355640905965218240	1.0797298075131560120893	0.99609254459969210644037
	0.8	1.0610194419890890932426	1.0640758462852161639507	0.99205120133814574312370
	0.9	1.0449476081320856577175	1.0459466997980809258315	0.98852708727010639205272
	$1-10^{-3}$	1.0254296331624704143181	1.0254350423202215142279	0.98663448912358955419566
	$1-10^{-6}$	1.0252183798772315195570	1.0252183852434649055998	0.98663269623852622075670
	$1-10^{-9}$	1.0252181685350061825380	1.0252181685403723730335	0.98663269470776782622584
	$1-10^{-12}$	1.0252181683236638691405	1.0252181683236692353310	0.98663269470623733124815
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	0.98663269470623579922141
He:Xe	10^{-100}	1.0252181683234523152732	1.1600604943154697563118	1.0252181683234523152732
	10^{-12}	1.0252181683240983261697	1.1600604943154041605928	1.0252181683234217174629
	10^{-9}	1.0252181689694632091047	1.1600604942498740373000	1.0252181682928545049959
	10^{-6}	1.0252188143315938446937	1.1600604287197234051675	1.0252181377256334210342
	10^{-3}	1.0258614343863045752151	1.1599948712122167388828	1.0251875618851580944861
	0.1	1.0697183185162260545670	1.1532086699382970544808	1.0220696675084938584613
	0.2	1.0909460489534795397190	1.1456884397981849756138	1.0187338026571338247504
	0.3	1.1007028446396204705150	1.1373473217269767208485	1.0151959481886851318287
	0.4	1.1035361475640274222335	1.1279852846872190693664	1.0114430873481564263156
	0.5	1.1014261439483445862625	1.1173331283160122966834	1.0074668556459334244195
	0.6	1.0951928411297615309132	1.1050187845418018023967	1.0032703461971770278517
	0.7	1.0849874817193250325606	1.0905145920161403317009	0.99888486638129420204698
	0.8	1.0704532915363226700230	1.0730602838006018835117	0.99441812705671756483310
	0.9	1.0507810146504920943091	1.0516001920064291917283	0.99022919778069690978649
	$1-10^{-3}$	1.0254993341393084111803	1.0255032106502240916017	0.98783350426027473794442
	$1-10^{-6}$	1.0252184496491771301925	1.0252184534849037797964	0.98783362559713846060736
	$1-10^{-9}$	1.0252181686047781980601	1.0252181686086138839574	0.98783362613732749860145
	$1-10^{-12}$	1.0252181683237336411561	1.0252181683237374768420	0.98783362613786810932834
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	0.98783362613786865048064
Ne:Ar	10^{-100}	1.0252181683234523152732	1.0511849005743001404009	1.0252181683234523152732
	10^{-12}	1.0252181683234700489208	1.0511849005742811495837	1.0252181683234304177754
	10^{-9}	1.0252181683411859628887	1.0511849005553093231415	1.0252181683015548175033
	10^{-6}	1.0252181860570864391973	1.0511848815834785473563	1.0252181464259538404935
	10^{-3}	1.0252358884620685619511	1.0511659054171156347499	1.0251962701195499193437
	0.1	1.0268525698676256123799	1.0492404315562688547083	1.0230208268773704010553
	0.2	1.0281934714488914593858	1.0471966892507730380465	1.0208062046614268618256
	0.3	1.0292173802079323026769	1.0450397829003468895067	1.0185713208849180743994
	0.4	1.0299000812293699222141	1.0427541353805882151620	1.0163135715653145370794
	0.5	1.0302159491561190718612	1.0403222810501080413347	1.0140310240938126932874
	0.6	1.0301372619639991566582	1.0377246925279724722234	1.0117228956134842380588
	0.7	1.0296335518883510762993	1.0349396984806199935670	1.0093903388093634527008
	0.8	1.0286710816170667750579	1.0319436197075000280933	1.0070377546951421027383
	0.9	1.0272126482680727259898	1.0287113846178620475448	1.0046750358043694053391
	$1-10^{-3}$	1.0252409015804517672357	1.0252544669421325526147	1.0023449012875361237734
	$1-10^{-6}$	1.0252181910855254369741	1.0252182046361879581283	1.0023215339660112140325

Table 2 (*continued*)

Mixture	x_1	$[\lambda]^\star_{70}$	$a_1^{(70)\star}$	$a_{-1}^{(70)\star}$
	$1-10^{-9}$	1.0252181683462144172176	1.0252181683597650650359	1.0023215106008177036756
	$1-10^{-12}$	1.0252181683234750773751	1.0252181683234886280229	1.0023215105774525123010
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	1.0023215105774291237211
Ne:Kr	10^{-100}	1.0252181683234523152732	1.0834970858539689321932	1.0252181683234523152732
	10^{-12}	1.0252181683235235473900	1.0834970858539338422908	1.0252181683234086794023
	10^{-9}	1.0252181683946844320102	1.0834970858188790297823	1.0252181682798164444505
	10^{-6}	1.0252182395555126929495	1.0834970507640544322024	1.0252181246875865529654
	10^{-3}	1.0252893439991010053224	1.0834619838428602411258	1.0251745375159390569475
	0.1	1.0317538034457581865657	1.0798593318451112648560	1.0209031894616197622739
	0.2	1.0370345719623078520598	1.0759307850795801279269	1.0166780294697338293328
	0.3	1.0409660311232228314890	1.0716557746400348147475	1.0125338454585848687219
	0.4	1.0434768553555725049437	1.0669695701641027957819	1.0084660872191902251257
	0.5	1.0445071694319858267100	1.0617965303722640538631	1.0044771586048062098147
	0.6	1.0439985955546405953179	1.0560483353527281298357	1.0005812819281029839658
	0.7	1.0418865685412835500502	1.0496231645165848694475	0.99681371307870693217135
	0.8	1.0380969495951092042679	1.0424083202537382342370	0.99324911861411916536843
	0.9	1.0325535579969326603521	1.0342936077679115819449	0.99004084396319170086428
	$1-10^{-3}$	1.0253002118644787144220	1.0253135261395093456095	0.98753298118065695024068
	$1-10^{-6}$	1.0252182504515713424560	1.0252182637245393961487	0.98751294878502560889172
	$1-10^{-9}$	1.0252181684055805188295	1.0252181684188534455741	0.98751292881927830419322
	$1-10^{-12}$	1.0252181683235344434768	1.0252181683235477164035	0.98751292879931262370803
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	0.98751292879929263804194
Ne:Xe	10^{-100}	1.0252181683234523152732	1.1069442666657445059470	1.0252181683234523152732
	10^{-12}	1.0252181683236047194143	1.1069442666657026283857	1.0252181683234048229137
	10^{-9}	1.0252181684758564561564	1.1069442666238669446346	1.0252181682759599557717
	10^{-6}	1.0252183207273865272012	1.1069442247881645174084	1.0252181208310945356915
	10^{-3}	1.0253703657116489256021	1.1069023703986860004890	1.0251706776862948248048
	0.1	1.0385045278591932943301	1.1025580717125662125460	1.0204849143062418182369
	0.2	1.0482664015249918505376	1.0977238100945130293158	1.0157789797086240241436
	0.3	1.0549389819172681048420	1.0923508247729991721718	1.0110948357866456113386
	0.4	1.0588074265852574894028	1.0863253317536433674199	1.0064306386127324797770
	0.5	1.0600358675779038833495	1.0795029498732825055427	1.0017923472389666478727
	0.6	1.0586815011033986612090	1.0716988468471293416856	0.99720238378238453685954
	0.7	1.0546984476142186419322	1.0626758782500665472169	0.99271873257619682214229
	0.8	1.0479379936695128561528	1.0521350805908460982369	0.98848023882267819845147
	0.9	1.0381679767576370151470	1.0397306735775055347933	0.98482605566575273007810
	$1-10^{-3}$	1.0253627623476950184478	1.0253732060895206825256	0.98267206364278991840615
	$1-10^{-6}$	1.0252183130551717891806	1.0252183234466689239379	0.98266413828441624981952
	$1-10^{-9}$	1.0252181684681841721866	1.0252181684785756170737	0.98266413055168090787292
	$1-10^{-12}$	1.0252181683235970471302	1.0252181683236074385750	0.98266413054394836588636
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	0.98266413054394062560428
Ar:Kr	10^{-100}	1.0252181683234523152732	1.0499416928603626880016	1.0252181683234523152732
	10^{-12}	1.0252181683234604898855	1.0499416928603408062417	1.0252181683234265562337
	10^{-9}	1.0252181683316269276020	1.0499416928384809281629	1.0252181682976932757685
	10^{-6}	1.0252181764980666843098	1.0499416709786017725410	1.0252181425644175992581
	10^{-3}	1.0252263449635295230630	1.0499198100203548164954	1.0251924140751367712450
	0.1	1.0260419721827594171549	1.0477404842606796749271	1.0226880836183203723558
	0.2	1.0268257874276644760644	1.0455046722798258147883	1.0202419065248053380918
	0.3	1.0274987585311430258008	1.0432224483751893324191	1.0178696087605757580455
	0.4	1.0280006360151590403224	1.0408832360879244358038	1.0155631748808700746587
	0.5	1.0282812225666775709259	1.0384776279886286895840	1.0133165820087948663588
	0.6	1.0282996460522037112554	1.0359974042848560268605	1.0111258818999988218598
	0.7	1.0280235657720448760889	1.0334356506346579820971	1.0089894060119765025120
	0.8	1.0274284177249907781396	1.0307870053788668738136	1.0069081284583720468261
	0.9	1.0264968070980921392752	1.0280480852908589567710	1.0048862426782185168334
	$1-10^{-3}$	1.0252326904595083265659	1.0252469118477304239615	1.0029512072391133774967
	$1-10^{-6}$	1.0252181828631363017787	1.0252181970713945653581	1.0029320590815687625233
	$1-10^{-9}$	1.0252181683379920168076	1.0252181683522002619404	1.0029320399374436795476
	$1-10^{-12}$	1.0252181683234668549747	1.0252181683234810632198	1.0029320399182995584999
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	1.0029320399182803952155
Ar:Xe	10^{-100}	1.0252181683234523152732	1.0700201810234063523819	1.0252181683234523152732
	10^{-12}	1.0252181683234869262026	1.0700201810233736915572	1.0252181683234120328723
	10^{-9}	1.0252181683580632446783	1.0700201809907455276399	1.0252181682831699143855
	10^{-6}	1.0252182029343728047575	1.0700201483625750635481	1.0252181280410604589834
	10^{-3}	1.0252527702850594798073	1.0699875136396837808084	1.0251778949589419211223
	0.1	1.0285488780211025920802	1.0666829633245630399912	1.0212764574138503188608
	0.2	1.0314708146480408198614	1.0631810615562466889714	1.0174938064079498908608
	0.3	1.0337974098516583661098	1.0594806562510814286273	1.0138526946955889653890
	0.4	1.0353850209689786282648	1.0555474343519695013500	1.0103406420510440749626
	0.5	1.0361264682412749310272	1.0513465204139416131429	1.0069511977512331724299
	0.6	1.0359443248669038204784	1.0468428429681176301363	1.0036857853293353794902
	0.7	1.0347850603709655614027	1.0420023195974687457325	1.0005569596542727333786

(*continued on next page*)

Table 2 (*continued*)

Mixture	x_1	$[\lambda]^\star_{70}$	$a_1^{(70)\star}$	$a_{-1}^{(70)\star}$
	0.8	1.0326152375828588130177	1.0367946382409654692024	0.9975941298881666863544
	0.9	1.0294215485323289035758	1.0311992575538279809806	0.9948538263291410664799
	$1-10^{-3}$	1.0252650502799158537620	1.0252797807776680327943	0.9924608638123646351592
	$1-10^{-6}$	1.0252182152530562002857	1.0252182299530063074141	0.9924388608991560251276
	$1-10^{-9}$	1.0252181683703819667894	1.0252181683850818863507	0.9924388389211473014929
	$1-10^{-12}$	1.0252181683234992449247	1.0252181683235139448443	0.9924388388991693177139
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	0.9924388388991473177301
Kr:Xe	10^{-100}	1.0252181683234523152732	1.0409279615964258769814	1.0252181683234523152732
	10^{-12}	1.0252181683234577755507	1.0409279615964119936008	1.0252181683234375334128
	10^{-9}	1.0252181683289125928245	1.0409279615825424962921	1.0252181683086704549553
	10^{-6}	1.0252181737837262608345	1.0409279477130438960172	1.0252181535415921143916
	10^{-3}	1.0252236249894290960719	1.0409140769222691362274	1.0252033865799581220083
	0.1	1.0257259902623583814072	1.0395261683042128121407	1.0237408824255047782942
	0.2	1.0261492215722242203065	1.0380953482158022889977	1.0222643543166986275567
	0.3	1.0264761637754997148820	1.0366322984109017011181	1.0207871223420528618654
	0.4	1.0266958232092931551911	1.0351337834347892040098	1.0193078830244211399245
	0.5	1.0267978427749659568629	1.0335965476291205115021	1.0178255272925145258106
	0.6	1.0267724375701266740052	1.0320173365526416378808	1.0163391899441820818645
	0.7	1.0266103384997581480921	1.0303929311197089008589	1.0148483168561982709987
	0.8	1.0263027490834861141157	1.0287201996203858479440	1.0133527562223649574479
	0.9	1.0258413224542803786291	1.0269961750289966695083	1.0118528826549812676516
	$1-10^{-3}$	1.0252252251219746809347	1.0252362234477554823463	1.0103648082707662297271
	$1-10^{-6}$	1.0252181753886992495033	1.0252181863813876666944	1.0103497817565262524075
	$1-10^{-9}$	1.0252181683305175706570	1.0252181683415102534360	1.0103497667299443711960
	$1-10^{-12}$	1.0252181683234593805286	1.0252181683234703732113	1.0103497667149177892476
	$1-10^{-100}$	1.0252181683234523152732	1.0252181683234523152732	1.0103497667149027476240

Table 3
Order 1 and normalized order 70 values of the diffusion coefficients, $[D_{12}]_1$ and $[D_{12}]^\star_{70}$, for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe as functions of the mixture fractions of the lighter constituents.

Mixture	x_1	$[D_{12}]_1 \times 10^1$ (cm^2 s^{-1})	$d_0^{(1)} \times 10^4$ (cm)	$[D_{12}]^\star_{70}$
He:Ne	10^{-100}	8.4763042862904107387921	$3.9115816397097755878377 \times 10^{99}$	1.0183168264410251895622
	10^{-12}	8.4763042862901931669741	$3.9115816397135867660586 \times 10^{11}$	1.0183168264410047015490
	10^{-9}	8.4763042860728389206356	$3.9115816435209538125484 \times 10^{8}$	1.0183168264205371764332
	10^{-6}	8.4763040687184585154558	$3.9115854508917458012051 \times 10^{5}$	1.0183168059530133867482
	10^{-3}	8.4760865802107368829116	$3.9153965709038662611980 \times 10^{2}$	1.0182963397550554038402
	0.1	8.4531409327259520842838	4.3343248757610913389940	1.0162810210604553212885
	0.2	8.4268774935701213553319	2.4304828326867268473977	1.0142707324666846504787
	0.3	8.3970002169441953738709	1.8452309474611796647776	1.0122868360732286968016
	0.4	8.3628585556154733300392	1.6080123127552058405632	1.0103326982685724016705
	0.5	8.3236193007673914461865	1.5364486848578973149314	1.0084148710654886633535
	0.6	8.2782004116703452536404	1.5917342259104439664689	1.0065441532326442371686
	0.7	8.2251733349379864053953	1.8074721917041147115100	1.0047371827722611333787
	0.8	8.1626149055646097075148	2.3542641284325308638735	1.0030188170227351023179
	0.9	8.0878757140117152567131	4.1470361346501712518512	1.0014256873072235915472
	$1-10^{-3}$	7.9982060829641790033697	$3.6946470961887923658574 \times 10^{2}$	1.0000244973007121061659
	$1-10^{-6}$	7.9972053694656941375862	$3.6904943377904185266778 \times 10^{5}$	1.0000114859182961044521
	$1-10^{-9}$	7.9972043676947105927527	$3.6904901886967740891136 \times 10^{8}$	1.0000114729196038577973
	$1-10^{-12}$	7.9972043666929385506321	$3.6904901845476841052619 \times 10^{11}$	1.0000114729066051782563
	$1-10^{-100}$	7.9972043666919357758141	$3.6904901845435308620385 \times 10^{99}$	1.0000114729065921665650
He:Ar	10^{-100}	5.5081421910969419213005	$3.4267671645201063191823 \times 10^{99}$	1.0276991471770545066307
	10^{-12}	5.5081421910968357900080	$3.4267671645234670591428 \times 10^{11}$	1.0276991471770320333917
	10^{-9}	5.5081421909908106287100	$3.4267671678808462829788 \times 10^{8}$	1.0276991471545812676404
	10^{-6}	5.5081420849655720494718	$3.4267705252633794193613 \times 10^{5}$	1.0276991247038110332178
	10^{-3}	5.5080359823909987338224	$3.4301312203754157133374 \times 10^{2}$	1.0276766694485398540667
	0.1	5.4966966008444349281257	3.7996072746327150899216	1.0254049904728501454123
	0.2	5.4833120202466383740404	2.1320747690313311977752	1.0230092014092613142592
	0.3	5.4674673638193076080134	1.6197439203388347247043	1.0204996876698305021743
	0.4	5.4484375250491467956195	1.4123430188309547098572	1.0178647620804553660568
	0.5	5.4251827795727345402285	1.3500623306790428807933	1.0150946599494570970895
	0.6	5.3961575055509437075705	1.3987910013538033525692	1.0121848343357205178638
	0.7	5.3589600474783232417197	1.5875984946306474422090	1.0091430858738213422810
	0.8	5.3096450802455310227565	2.0645478984786347835705	1.0060053800336177258716
	0.9	5.2412462929357231707512	3.6230265173669789336349	1.0028725699954269374287
	$1-10^{-3}$	5.1414489800560342565034	$3.2018390440510426892361 \times 10^{2}$	1.0000267165334709567652
	$1-10^{-6}$	5.1402021985328269393394	$3.1978647457015279591075 \times 10^{5}$	1.0000009119184908803673
	$1-10^{-9}$	5.1402009487862549630523	$3.1978607735328342103365 \times 10^{8}$	1.0000008861553395349888
	$1-10^{-12}$	5.1402009475365054190069	$3.1978607695606676397463 \times 10^{11}$	1.0000008861295764252605
	$1-10^{-100}$	5.1402009475352544184593	$3.1978607695566914970351 \times 10^{99}$	1.0000008861295506363619
He:Kr	10^{-100}	4.5306839945775141716672	$3.9839103673238034196874 \times 10^{99}$	1.0352324218241692533825
	10^{-12}	4.5306839945774379824594	$3.9839103673277203355402 \times 10^{11}$	1.0352324218241437907153

Table 3 (*continued*)

Mixture	x_1	$[D_{12}]_1 \times 10^1$ (cm^2 s^{-1})	$d_0^{(1)} \times 10^4$ (cm)	$[D_{12}]^\star_{70}$
	10^{-9}	4.5306839945013249638514	$3.9839103712407192763562 \times 10^8$	1.0352324217987065861326
	10^{-6}	4.5306839183882477336115	$3.9839142842435215436909 \times 10^5$	1.0352323963614957403356
	10^{-3}	4.5306077466437591665714	$3.9878311523227998917670 \times 10^2$	1.0352069528838151493088
	0.1	4.5224294706644430828242	4.4185022431330002073029	1.0326195960855772365625
	0.2	4.5126732316230936372122	2.4800457410686601925796	1.0298573546739326738725
	0.3	4.5009663047801235719219	1.8846567040008428874896	1.0269191328237492534774
	0.4	4.4866613488719992222545	1.6438335326258087657426	1.0237753270927140609870
	0.5	4.4687896214464131817006	1.5717942212325520910355	1.0203937309226589043885
	0.6	4.4458334607219451188444	1.6288749149837110029094	1.0167420028902002829000
	0.7	4.4152737153708521473472	1.8487752727308826249823	1.0127956318669021361727
	0.8	4.3726011924052385896215	2.4030658565357625614472	1.0085614270339883835165
	0.9	4.3088695620171097665509	4.2098500261061897607213	1.0041483867252117467250
	$1-10^{-3}$	4.2048545830459509390522	$3.7011039213623684187090 \times 10^2$	1.0000367489069942793940
	$1-10^{-6}$	4.2034620930733688049443	$3.6961820726237055292757 \times 10^5$	1.0000000864789661374503
	$1-10^{-9}$	4.2034606960361018900969	$3.6961771516982945534355 \times 10^8$	1.0000000498959049427857
	$1-10^{-12}$	4.2034606946390600610566	$3.6961771467773700518228 \times 10^{11}$	1.0000000498593219613757
	$1-10^{-100}$	4.2034606946376616207827	$3.6961771467724442014717 \times 10^{99}$	1.0000000498592853417748
He:Xe	10^{-100}	3.6157493449606025456659	$3.9467279906441958789397 \times 10^{99}$	1.0382919333040952019429
	10^{-12}	3.6157493449605533046527	$3.9467279906480888584908 \times 10^{11}$	1.0382919333040723355411
	10^{-9}	3.6157493449113615324083	$3.9467279945371754338671 \times 10^8$	1.0382919332812288000784
	10^{-6}	3.6157492957195491076837	$3.9467318836275960384128 \times 10^5$	1.0382919104376852094024
	10^{-3}	3.6157000636939913897324	$3.9506248230896087193485 \times 10^2$	1.0382690587612933982572
	0.1	3.6103872901727786145158	4.3787501153500986743217	1.0359188682522738769257
	0.2	3.6039784536533104914039	2.4586747593096589468336	1.0333506360869783803579
	0.3	3.5961834647925582536784	1.8692243275960435012124	1.0305482693651365020972
	0.4	3.5864985888342557930929	1.6311665322208495486021	1.0274637424145147932546
	0.5	3.5741425664176804106484	1.5605250434844225082703	1.0240375819521488243020
	0.6	3.5578350699673838735157	1.6181301482621653621329	1.0201969832418349789004
	0.7	3.5353226939598305423751	1.8375901146726645742585	1.0158577081444922393307
	0.8	3.5022382087726656527899	2.3892664164710787154947	1.0109426497327035504026
	0.9	3.4488641724603856983048	4.1828516386852860799964	1.0054718673523624506821
	$1-10^{-3}$	3.3497713350520273034966	$3.6600629352010788230593 \times 10^2$	1.0000486313851520143937
	$1-10^{-6}$	3.3483298348705944357267	$3.6548330748012956193090 \times 10^5$	1.0000000569977423060979
	$1-10^{-9}$	3.3483283870060813690063	$3.6548278432238408607535 \times 10^8$	1.0000000085509241046406
	$1-10^{-12}$	3.3483283855582104635234	$3.6548278379922616600016 \times 10^{11}$	1.0000000085024774149925
	$1-10^{-100}$	3.3483283855567611432913	$3.6548278379870248439831 \times 10^{99}$	1.0000000085024289198078
Ne:Ar	10^{-100}	2.4227813806350481562314	$1.7629847801317523289545 \times 10^{99}$	1.0062522721826468862490
	10^{-12}	2.4227813806350152192704	$1.7629847801334913465037 \times 10^{11}$	1.0062522721826422164967
	10^{-9}	2.4227813806021111952472	$1.7629847818707698798638 \times 10^8$	1.0062522721779771338901
	10^{-6}	2.4227813476980680012377	$1.7629865191510265684592 \times 10^5$	1.0062522675128933582411
	10^{-3}	2.4227484244749197300241	$1.7647255244546084791023 \times 10^2$	1.0062476012598592175778
	0.1	2.4192861674295634262605	1.9560460212531220589388	1.0057733721101207241714
	0.2	2.4153453450163359123453	1.0984836261076856935115	1.0052697314261031008192
	0.3	2.4108842485460844608998	0.83539409367199932084619	1.0047400410478414128059
	0.4	2.4058103696003697360989	0.72943145348523284961258	1.0041830960307227267830
	0.5	2.4000076900221865959484	0.69856522153050772476972	1.0035979230487959601402
	0.6	2.3933287101359966110835	0.72564706751697477918656	1.0029839853027986983600
	0.7	2.3855829971953904334319	0.82662697183542539972465	1.0023415123293584777337
	0.8	2.3765203428992054154501	1.0808262632807663681958	1.0016720343864484973468
	0.9	2.3658053170877858057442	1.9128055787074538866555	1.0009792579461158030719
	$1-10^{-3}$	2.3531182791363715129719	$1.7140070538494521131677 \times 10^2$	1.0002776505333489962426
	$1-10^{-6}$	2.3529777535143478428812	$1.7121925027404610762541 \times 10^5$	1.0002705286485104167135
	$1-10^{-9}$	2.3529776128520878869209	$1.7121906899044740880384 \times 10^8$	1.0002705215264551998259
	$1-10^{-12}$	2.3529776127114254902000	$1.7121906880916398143646 \times 10^{11}$	1.0002705215193331444415
	$1-10^{-100}$	2.3529776127112846869999	$1.7121906880898251654437 \times 10^{99}$	1.0002705215193260152569
Ne:Kr	10^{-100}	1.9017257940137846498686	$1.8197638784499995131832 \times 10^{99}$	1.0155056865684927475768
	10^{-12}	1.9017257940137524155837	$1.8197638784517884320325 \times 10^{11}$	1.0155056865684809368274
	10^{-9}	1.9017257939815503648879	$1.8197638802389183643226 \times 10^8$	1.0155056865566819981913
	10^{-6}	1.9017257617794756953895	$1.8197656673706148438017 \times 10^5$	1.0155056747577401142809
	10^{-3}	1.9016935357157592616782	$1.8215545650077942492129 \times 10^2$	1.0154938725671789368343
	0.1	1.8982460099217917474904	2.0182600761553190330141	1.0142911781353565677076
	0.2	1.8941783153068666104721	1.1328385539298615394765	1.0130060135002422162116
	0.3	1.8893829001436765890578	0.86092997490448906927494	1.0116444068074660293919
	0.4	1.8836714489914590681657	0.75103652178552485290356	1.0102007770729167771179
	0.5	1.8767841159918679468663	0.71835886176258269892850	1.0086705244075222340047
	0.6	1.8683520081831108293757	0.74492852479562448812354	1.0070515599994285398204
	0.7	1.8578332890404280886930	0.84655384929580455729692	1.0053473318970966428332
	0.8	1.8443983891744594938648	1.1030670065106654173376	1.0035729243629649068061
	0.9	1.8267097938606840552114	1.9422010784697118819874	1.0017677494902959664937
	$1-10^{-3}$	1.8027496130448371347839	$1.7267802227512430255923 \times 10^2$	1.0000396945720376302292
	$1-10^{-6}$	1.8024633212551240126212	$1.7247812143248663979317 \times 10^5$	1.0000230929570504719276
	$1-10^{-9}$	1.8024630344377952628299	$1.7247792168125574644605 \times 10^8$	1.0000230763676910509640
	$1-10^{-12}$	1.8024630341509774075915	$1.7247792148150466492398 \times 10^{11}$	1.0000230763511017038458

(*continued on next page*)

Table 3 (*continued*)

Mixture	x_1	$[D_{12}]_1 \times 10^1$ (cm^2 s^{-1})	$d_0^{(1)} \times 10^4$ (cm)	$[D_{12}]^\star_{70}$
	$1-10^{-100}$	1.802463034150690302630 8	$1.7247792148130471389157 \times 10^{99}$	1.0000230763510850978927
Ne:Xe	10^{-100}	1.5043432550153760957201	$1.7374454032581225472695 \times 10^{99}$	1.0219922700092599257805
	10^{-12}	1.5043432550153529500433	$1.7374454032598332605094 \times 10^{11}$	1.0219922700092454441096
	10^{-9}	1.5043432549922304189428	$1.7374454049688357888256 \times 10^{8}$	1.0219922699947782549137
	10^{-6}	1.5043432318696811744195	$1.7374471139730521520556 \times 10^{5}$	1.0219922555275837799641
	10^{-3}	1.5043200911629506039033	$1.7391578079106051986861 \times 10^{2}$	1.0219777830519380255104
	0.1	1.5018325274673740506511	1.9272729239245678338560	1.0204890428958482691599
	0.2	1.4988613058995203293079	1.0819462568526167668132	1.0188661366760817418686
	0.3	1.4952984088022948117984	0.8223804919557587427321 8	1.0171075734120072255844
	0.4	1.4909580625651764475268	0.7174942242564692778873 3	1.0151951678385277499543
	0.5	1.4855682405468183320718	0.6863044593470642890147 8	1.0131089230886696986 9
	0.6	1.4787140801513032450678	0.7116020486920289638665 1	1.0108286726184741561896
	0.7	1.4697300353273813878580	0.8083184616392481801913 9	1.0083387403572518114294
	0.8	1.4574769403500588257929	1.0520731396921275350828	1.0056409566784675743361
	0.9	1.4398297029205057016961	1.8477058864749671954089	1.0027907468052454654187
	$1-10^{-3}$	1.4126671980318517895297	$1.6331970847620366706341 \times 10^{2}$	1.0000304071965164664379
	$1-10^{-6}$	1.4123168104127010562574	$1.6311608376882335488071 \times 10^{5}$	1.0000045033143773098855
	$1-10^{-9}$	1.4123164590514489666378	$1.6311588023529047284051 \times 10^{8}$	1.0000044774439356508956
	$1-10^{-12}$	1.4123164587000867382266	$1.6311588003175703073128 \times 10^{11}$	1.0000044774180652428604
	$1-10^{-100}$	1.4123164586997350242833	$1.6311588003155329355208 \times 10^{99}$	1.0000044774180393465561
Ar:Kr	10^{-100}	1.0846367457840479979186	$1.1322620158308577512248 \times 10^{99}$	1.0068324667203306894611
	10^{-12}	1.0846367457840285717450	$1.1322620158319697340841 \times 10^{11}$	1.0068324667203241335750
	10^{-9}	1.0846367457646218243458	$1.1322620169428406115423 \times 10^{8}$	1.0068324667137748032859
	10^{-6}	1.0846367263578647124300	$1.1322631278148188014812 \times 10^{5}$	1.0068324601644441657352
	10^{-3}	1.0846173098853313001100	$1.1333751016236912033052 \times 10^{2}$	1.0068259104855013374309
	0.1	1.0825940208931575130212	1.2556995522412789312494	1.0061735674236652710163
	0.2	1.0803385364888014556582	0.7048594229008090729284 3	1.0055088149612418032894
	0.3	1.0778489540844345305070	0.5357981806028661956878 2	1.0048395242421551082013
	0.4	1.0751000520238225736752	0.4676277399076787850634 9	1.0041672916690566385397
	0.5	1.0720619296385905476128	0.4476540210411750790683 6	1.0034940900571199587042
	0.6	1.0686989913198462764226	0.4648435213185553571871 5	1.0028223857488848750847
	0.7	1.0649686387138605981237	0.5293953822191104436015 8	1.0021552856317673839536
	0.8	1.0608195707249873474782	0.6921244083852500642767 2	1.0014967240765948297276
	0.9	1.0561895491536178869290	1.2250730360213268675642	1.0008517028912299836583
	$1-10^{-3}$	1.0510573457890158894272	$1.0983064858349848813676 \times 10^{2}$	1.0002327286743526543410
	$1-10^{-6}$	1.0510024802482199386110	$1.0971520018715923874620 \times 10^{5}$	1.0002266074649812343749
	$1-10^{-9}$	1.0510024253502667216906	$1.0971508485082761883748 \times 10^{8}$	1.0002266013451633539675
	$1-10^{-12}$	1.0510024252953687360439	$1.0971508473549139916842 \times 10^{11}$	1.0002266013390435374801
	$1-10^{-100}$	1.0510024252953137831053	$1.0971508473537594749719 \times 10^{99}$	1.0002266013390374115377
Ar:Xe	10^{-100}	0.8621653018592700268679 4	$1.0587452089497119229562 \times 10^{99}$	1.0122348497012336614750
	10^{-12}	0.8621653018592534059914 0	$1.0587452089507502576089 \times 10^{11}$	1.0122348497012226122126
	10^{-9}	0.8621653018426491503157 9	$1.0587452099880465766621 \times 10^{8}$	1.0122348496901843991290
	10^{-6}	0.8621652852383823440449 3	$1.0587462472853892576279 \times 10^{5}$	1.0122348386519697467223
	10^{-3}	0.8621486698354173422466 8	$1.0597845692737380350603 \times 10^{2}$	1.0122237988684615845381
	0.1	0.8603859787901368504736 7	1.1739557637640628807398	1.0111141344787978646606
	0.2	0.8583460803903870073663 3	0.6587844859001467887049 1	1.0099616503843314104316
	0.3	0.8559988683437720540181 1	0.5005584686276185694040 7	1.0087775859371034899076
	0.4	0.8532847793826747236620 0	0.4365999430411341168992 3	1.0075630752624000603203
	0.5	0.8501267988370502650008 5	0.4175847361648517001242 1	1.0063207555412913215961
	0.6	0.8464236647690731628904 3	0.4330893187784297810252 6	1.0050556258810590312079
	0.7	0.8420396167380101702315 0	0.4923955821269770501156 0	1.0037763936225805413609
	0.8	0.8367884512507731792503 3	0.6422389083593169395197 6	1.0024976231156016213284
	0.9	0.8304078203049450605094 6	1.1330519917265275682919	1.0012432298262261105002
	$1-10^{-3}$	0.8226040952472193483689 4	$1.0111749368363685961300 \times 10^{2}$	1.0000636769289761947249
	$1-10^{-6}$	0.8225161062793619120639 0	$1.0100567208691140123687 \times 10^{5}$	1.0000522751552412447238
	$1-10^{-9}$	0.8225160181875225810703 4	$1.0100556036450346963386 \times 10^{8}$	1.0000522637596478875179
	$1-10^{-12}$	0.8225160180994306387515 3	$1.0100556025278116077758 \times 10^{11}$	1.0000522637482523003543
	$1-10^{-100}$	0.8225160180993424586289 8	$1.0100556025266932663468 \times 10^{99}$	1.0000522637482408933601
Kr:Xe	10^{-100}	0.5783515361081200479080 9	$0.7959761454985416270655 7 \times 10^{99}$	1.0042215316500265988517
	10^{-12}	0.5783515361081127509638 3	$0.7959761454993275605411 7 \times 10^{11}$	1.0042215316500232384496
	10^{-9}	0.5783515361008231036486 8	$0.7959761462844751034461 9 \times 10^{8}$	1.0042215316466661967437
	10^{-6}	0.5783515288111729921272 4	$0.7959769314327993074058 6 \times 10^{5}$	1.0042215282896241159536
	10^{-3}	0.5783442363638916294060 6	$0.7967628618369212724253 4 \times 10^{2}$	1.0042181708726392850554
	0.1	0.5775930269164712142003 6	0.8832580235535350398294 5	1.0038817543497835388810
	0.2	0.5767733803039593583133 7	0.4961275965154131346140 3	1.0035345845711090259847
	0.3	0.5758868006175514258999 8	0.3774209374817321758277 6	1.0031801931507350181548
	0.4	0.5749266827021224046500 7	0.3296927389532248597213 8	1.0028188299248630876719
	0.5	0.5738854679016506140305 3	0.3159318263573083892052 2	1.0024508458219528004037
	0.6	0.5727544680622789775264 0	0.3284470785659936643729 7	1.0020767206319061451381
	0.7	0.5715236489304424409907 3	0.3745614435701547791608 9	1.0016970980640842120146
	0.8	0.5701813615872482330476 4	0.4904572890536107788651 8	1.0013128302065296455202
	0.9	0.5687140067771615878499 7	0.8696801834241167280994 0	1.0009250341669299405469

Table 3 (*continued*)

Mixture	x_1	$[D_{12}]_1 \times 10^1$ $(\mathrm{cm^2\,s^{-1}})$	$d_0^{(1)} \times 10^4$ (cm)	$[D_{12}]^\star_{70}$
	$1-10^{-3}$	0.56712245310724545866441	$0.78130303777315495035907 \times 10^{2}$	1.0005390677522271054163
	$1-10^{-6}$	0.56710562857622933553051	$0.78049935989666106354738 \times 10^{5}$	1.0005351684743968558690
	$1-10^{-9}$	0.56710561174372837500463	$0.78049855701149273382672 \times 10^{8}$	1.0005351645751069970858
	$1-10^{-12}$	0.56710561172689586607075	$0.78049855620860835740776 \times 10^{11}$	1.0005351645712077072153
	$1-10^{-100}$	0.56710561172687901671245	$0.78049855620780466934407 \times 10^{99}$	1.0005351645712038040222

Table 4
Order 1 and normalized order 70 values of the thermal diffusion coefficients, $[D_T]_1$ and $[D_T]^\star_{70}$, for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe as functions of the mixture fractions of the lighter constituents.

Mixture	x_1	$[D_T]_1 \times 10^3$ $(\mathrm{cm^2\,s^{-1}})$	$[D_T]^\star_{70}$
He:Ne	10^{-100}	$-31.646907680137460472975 \times 10^{-99}$	1.1727641522135545606618
	10^{-12}	$-31.646907680118735403060 \times 10^{-11}$	1.1727641522134524088207
	10^{-9}	$-31.646907661412390552449 \times 10^{-8}$	1.1727641521114027195723
	10^{-6}	$-31.646888955062234709054 \times 10^{-5}$	1.1727640500616998308119
	10^{-3}	$-31.628177295602045582489 \times 10^{-2}$	1.1726619867110624152583
	0.1	−29.717047551859776833746	1.1624053508495331483336
	0.2	−55.307853254800154538740	1.1517334502359457509954
	0.3	−76.276452422536648304947	1.1407145035408151851090
	0.4	−91.969804904830071696131	1.1293204418630852362592
	0.5	−101.52871760112091088754	1.1175292487057323868719
	0.6	−103.81640112778701336482	1.1053264335986878282488
	0.7	−97.312443611781126866976	1.0927083471780871580618
	0.8	−79.951559889475667142950	1.0796884317964479207634
	0.9	−48.870939351579418155564	1.0663085295817498046438
	$1-10^{-3}$	$-59.432087778246384755913 \times 10^{-2}$	1.0527967781237656594892
	$1-10^{-6}$	$-59.549417204218899741047 \times 10^{-5}$	1.0526595763539228474802
	$1-10^{-9}$	$-59.549534652441843391280 \times 10^{-8}$	1.0526594391469324378142
	$1-10^{-12}$	$-59.549534769890185255684 \times 10^{-11}$	1.0526594390097254422230
	$1-10^{-100}$	$-59.549534770007751163575 \times 10^{-99}$	1.0526594390095880978830
He:Ar	10^{-100}	$-21.024214723838819088234 \times 10^{-99}$	1.2436199063947996916504
	10^{-12}	$-21.024214723832006010265 \times 10^{-11}$	1.2436199063946698748315
	10^{-9}	$-21.024214717025741114923 \times 10^{-8}$	1.2436199062649828727582
	10^{-6}	$-21.024207910756557429181 \times 10^{-5}$	1.2436197765779464721956
	10^{-3}	$-21.017397350275016965106 \times 10^{-2}$	1.2434900551961121412657
	0.1	−20.296803112963154068559	1.2302757015106409539546
	0.2	−38.924804936835662152211	1.2161249381753846915178
	0.3	−55.478803140513369449032	1.2010300503998733430707
	0.4	−69.397464365025487017262	1.1848290035165239530941
	0.5	−79.878876195090998587899	1.1673297086784892496578
	0.6	−85.734509791188947246065	1.1483046571310242587841
	0.7	−85.121921043933103320033	1.1274887704550815656798
	0.8	−75.019827346313033753815	1.1045900958343137867095
	0.9	−50.100399379168863940708	1.0793448926658645400110
	$1-10^{-3}$	$-68.368335046149426732652 \times 10^{-2}$	1.0520163604399200156974
	$1-10^{-6}$	$-68.596025644170986586974 \times 10^{-5}$	1.0517304388376960322879
	$1-10^{-9}$	$-68.596253870850547996913 \times 10^{-8}$	1.0517301528355986483420
	$1-10^{-12}$	$-68.596254099077764908375 \times 10^{-11}$	1.0517301525495964707777
	$1-10^{-100}$	$-68.596254099306220581497 \times 10^{-99}$	1.0517301525493101823116
He:Kr	10^{-100}	$-17.373920128273337679838 \times 10^{-99}$	1.2968000340186695863639
	10^{-12}	$-17.373920128269126222055 \times 10^{-11}$	1.2968000340185151612967
	10^{-9}	$-17.373920124061879893882 \times 10^{-8}$	1.2968000338642445191216
	10^{-6}	$-17.373915916812413297083 \times 10^{-5}$	1.2967998795935610511527
	10^{-3}	$-17.369705526550224988125 \times 10^{-2}$	1.2966455675893725297011
	0.1	−16.918791080573363020679	1.2809152871129083431000
	0.2	−32.767164503776283324354	1.2640206119339348769637
	0.3	−47.234238566642085418879	1.2459004765445057912311
	0.4	−59.873649233985846320998	1.2262918337576654091944
	0.5	−70.020031335205505282305	1.2048666354747263554431
	0.6	−76.634749531598722967630	1.1812088901466069346571
	0.7	−77.999317989894667953860	1.1547869365756260555096
	0.8	−71.046212886181635796939	1.1249315960961032779228
	0.9	−49.703189596737326174080	1.0908788522466058119317
	$1-10^{-3}$	$-72.779459697752492692542 \times 10^{-2}$	1.0525993069655127511565
	$1-10^{-6}$	$-73.088008365294018846840 \times 10^{-5}$	1.0521925154993775819198
	$1-10^{-9}$	$-73.088317917484597428414 \times 10^{-8}$	1.0521921085533173031530
	$1-10^{-12}$	$-73.088318227037794800448 \times 10^{-11}$	1.0521921081463710891510
	$1-10^{-100}$	$-73.088318227347657861888 \times 10^{-99}$	1.0521921081459637355833
He:Xe	10^{-100}	$-13.868513408059166448030 \times 10^{-99}$	1.3178604054407885105683
	10^{-12}	$-13.868513408056563427831 \times 10^{-11}$	1.3178604054406483056379
	10^{-9}	$-13.868513405456146246482 \times 10^{-8}$	1.3178604053005835800959
	10^{-6}	$-13.868510805036863320427 \times 10^{-5}$	1.3178602652358067635031

(*continued on next page*)

Table 4 (*continued*)

Mixture	x_1	$[D_T]_1 \times 10^3$ $(\mathrm{cm^2\,s^{-1}})$	$[D_T]^{\star}_{70}$
	10^{-3}	$-13.865908282472393554189 \times 10^{-2}$	1.3177201491517486058756
	0.1	−13.585318936706665988944	1.3032921381179493997273
	0.2	−26.494954485415725812075	1.2874738980629910572471
	0.3	−38.512229043697521940790	1.2701286271878396279516
	0.4	−49.316268543619856830080	1.2508987137962499940943
	0.5	−58.410583843902573472529	1.2293105810522523945894
	0.6	−64.983428142682521999497	1.2047202474504596798627
	0.7	−67.609366583029714982799	1.1762288998121947205052
	0.8	−63.531793565245638787476	1.1425582057315242545406
	0.9	−46.629015437677828719495	1.1019258784644679636786
	$1-10^{-3}$	$-74.172779147317950845888 \times 10^{-2}$	1.0530131859856985183114
	$1-10^{-6}$	$-74.573137142738955603498 \times 10^{-5}$	1.0524756587751760505820
	$1-10^{-9}$	$-74.573539264758720726561 \times 10^{-8}$	1.0524751208933209519528
	$1-10^{-12}$	$-74.573539666882512296013 \times 10^{-11}$	1.0524751203554387440760
	$1-10^{-100}$	$-74.573539667285038615674 \times 10^{-99}$	1.0524751203549003234471
Ne:Ar	10^{-100}	$-6.1604362857612546937808 \times 10^{-99}$	1.1044242646588764172807
	10^{-12}	$-6.1604362857578160408266 \times 10^{-11}$	1.1044242646588456076239
	10^{-9}	$-6.1604362823226017382092 \times 10^{-8}$	1.1044242646280667605213
	10^{-6}	$-6.1604328471068979964363 \times 10^{-5}$	1.1044242338492092542613
	10^{-3}	$-6.1569962295956009469345 \times 10^{-2}$	1.1043934445843824048919
	0.1	−5.8018275791320023419759	1.1012353408148853718646
	0.2	−10.821106443554966046832	1.0978142571966454698468
	0.3	−14.943117938804120319228	1.0941342546529624547375
	0.4	−18.025524536802534912697	1.0901649809186930353596
	0.5	−19.889602667258650702759	1.0858719380587339830258
	0.6	−20.307908059110321448339	1.0812158840267070325286
	0.7	−18.986556803145780952516	1.0761522347761726413915
	0.8	−15.539164985669461660031	1.0706305773621532571568
	0.9	−9.4474755683296669929697	1.0645945394961254298434
	$1-10^{-3}$	$-11.407227567462140488841 \times 10^{-2}$	1.0580517320233088746814
	$1-10^{-6}$	$-11.428799672666927294047 \times 10^{-5}$	1.0579826198431211967369
	$1-10^{-9}$	$-11.428821264409635135185 \times 10^{-8}$	1.0579825506989729518133
	$1-10^{-12}$	$-11.428821286001397498817 \times 10^{-11}$	1.0579825506298287715896
	$1-10^{-100}$	$-11.428821286023010874576 \times 10^{-99}$	1.0579825506297595581960
Ne:Kr	10^{-100}	$-6.7719126038873711499948 \times 10^{-99}$	1.1618007435374006822356
	10^{-12}	$-6.7719126038845914198987 \times 10^{-11}$	1.1618007435373355880057
	10^{-9}	$-6.7719126011076410525134 \times 10^{-8}$	1.1618007434723064523022
	10^{-6}	$-6.7719098241558672923531 \times 10^{-5}$	1.1618006784431444010917
	10^{-3}	$-6.7691314650864106831293 \times 10^{-2}$	1.1617356229217313257867
	0.1	−6.4788772927058113602607	1.1550154735750721302063
	0.2	−12.302396113057476069631	1.1476262724750767233438
	0.3	−17.341423514187456303009	1.1395450966939509555545
	0.4	−21.421578159689609398266	1.1306687705977410628483
	0.5	−24.301760726999090209598	1.1208758858944046120362
	0.6	−25.639171347371011332760	1.1100235818013765798020
	0.7	−24.929728881667926480077	1.0979451863147456201240
	0.8	−21.401034509024902592198	1.0844516171047693675273
	0.9	−13.807032130274600262743	1.0693449707586649821620
	$1-10^{-3}$	$-17.968289909741465715057 \times 10^{-2}$	1.0526475489940921617208
	$1-10^{-6}$	$-18.01777507091790552034 \times 10^{-5}$	1.0524700717950415806372
	$1-10^{-9}$	$-18.017827083209587445123 \times 10^{-8}$	1.0524698942297820013612
	$1-10^{-12}$	$-18.017827132785793922482 \times 10^{-11}$	1.0524698940522166537597
	$1-10^{-100}$	$-18.017827132835419754880 \times 10^{-99}$	1.0524698940520389106689
Ne:Xe	10^{-100}	$-5.6197609305460851494936 \times 10^{-99}$	1.2065913658590160409677
	10^{-12}	$-5.6197609305444088986999 \times 10^{-11}$	1.2065913658589339368023
	10^{-9}	$-5.6197609288698343547653 \times 10^{-8}$	1.2065913657769118755530
	10^{-6}	$-5.6197592542942216069185 \times 10^{-5}$	1.2065912837548134228850
	10^{-3}	$-5.6180836091018391918352 \times 10^{-2}$	1.2065092244342720173665
	0.1	−5.4406098857934376688820	1.1979887995719276879975
	0.2	−10.469062357741253546711	1.1885143303976847522322
	0.3	−14.982224290450673858566	1.1780137720913712183998
	0.4	−18.834808162857630214371	1.1662951941043735776819
	0.5	−21.814977284377278807580	1.1531170394730369685168
	0.6	−23.600753104059358711677	1.1381723588378715854792
	0.7	−23.676932837952786952728	1.1210692633784051686495
	0.8	−21.163353558091290059621	1.1013124150195424370659
	0.9	−14.420253091597100610090	1.0783113899880455156058
	$1-10^{-3}$	$-20.283558492747714365027 \times 10^{-2}$	1.0518275585004209956234
	$1-10^{-6}$	$-20.358959416892674487486 \times 10^{-5}$	1.0515411132567119723475
	$1-10^{-9}$	$-20.359035024970655189094 \times 10^{-8}$	1.0515408266304189888067
	$1-10^{-12}$	$-20.359035100578940895351 \times 10^{-11}$	1.0515408263437925150052
	$1-10^{-100}$	$-20.359035100654624865235 \times 10^{-99}$	1.0515408263435056016179

Table 4 (*continued*)

Mixture	x_1	$[D_T]_1 \times 10^3$ (cm^2 s^{-1})	$[D_T]^{\star}_{70}$
Ar:Kr	10^{-100}	$-2.8930139181149375335592 \times 10^{-99}$	1.1040628811513094072040
	10^{-12}	$-2.8930139181128090485410 \times 10^{-11}$	1.1040628811512728573301
	10^{-9}	$-2.8930139159864525149748 \times 10^{-8}$	1.1040628811147595333749
	10^{-6}	$-2.8930117896294879284803 \times 10^{-5}$	1.1040628446014284068086
	10^{-3}	$-2.8908850014148500368303 \times 10^{-2}$	1.1040263240968564290830
	0.1	$-2.6756143705243915345620$	1.1003340598632075619873
	0.2	$-4.8962185254238945660335$	1.0964496869794227464435
	0.3	$-6.6265399907344295433155$	1.0923985781039871202049
	0.4	$-7.8237711772878343641257$	1.0881703394320981103879
	0.5	$-8.4361378298315538775933$	1.0837553476619426452054
	0.6	$-8.4010560699362150277478$	1.0791448129716540287958
	0.7	$-7.6427461088314107142417$	1.0743309456609715822303
	0.8	$-6.0691178974129306830453$	1.0693072600999595279786
	0.9	$-3.5676665695744182038968$	1.0640690688828864201922
	$1-10^{-3}$	$-4.1486055147748695750741 \times 10^{-2}$	1.0586698686117168269653
	$1-10^{-6}$	$-4.1547677370503334044135 \times 10^{-5}$	1.0586143051789703141522
	$1-10^{-9}$	$-4.1547739024416099142778 \times 10^{-8}$	1.0586142496047743957235
	$1-10^{-12}$	$-4.1547739086070043615683 \times 10^{-11}$	1.0586142495492001890425
	$1-10^{-100}$	$-4.1547739086131759275847 \times 10^{-99}$	1.0586142495491445592060
Ar:Xe	10^{-100}	$-2.9235509152360107495408 \times 10^{-99}$	1.1380838897696135519887
	10^{-12}	$-2.9235509152344060806533 \times 10^{-11}$	1.1380838897695546336864
	10^{-9}	$-2.9235509136313418614504 \times 10^{-8}$	1.1380838897106952497644
	10^{-6}	$-2.9235493105665489868304 \times 10^{-5}$	1.1380838308512934936705
	10^{-3}	$-2.9219456717454862907689 \times 10^{-2}$	1.1380249536088664838076
	0.1	$-2.7569658387422710906811$	1.1320068654822395637967
	0.2	$-5.1529412930326866279899$	1.1255321292052965700150
	0.3	$-7.1372891832312021267627$	1.1186170876381376962719
	0.4	$-8.6443846631183837503990$	1.1112167411098189304275
	0.5	$-9.5883058105232508309726$	1.1032835885802823967408
	0.6	$-9.8550240927414346846155$	1.0947680242313181797283
	0.7	$-9.2906140106814243782108$	1.0856196775951882064190
	0.8	$-7.6829048893311175901515$	1.0757905940661632145697
	0.9	$-4.7318953441856082073082$	1.0652420837366393819172
	$1-10^{-3}$	$-5.8061454856897876107603 \times 10^{-2}$	1.0540755211786314075701
	$1-10^{-6}$	$-5.8181857428830523826394 \times 10^{-5}$	1.0539592320636190764725
	$1-10^{-9}$	$-5.8181977965087995549245 \times 10^{-8}$	1.0539591157393014302539
	$1-10^{-12}$	$-5.8181978085624386858775 \times 10^{-11}$	1.0539591156229770774178
	$1-10^{-100}$	$-5.8181978085745043907267 \times 10^{-99}$	1.0539591156228606366241
Kr:Xe	10^{-100}	$-1.0838227995845962484440 \times 10^{-99}$	1.0903902205754061427737
	10^{-12}	$-1.0838227995837953696045 \times 10^{-11}$	1.0903902205753835349901
	10^{-9}	$-1.0838227987837174087314 \times 10^{-8}$	1.0903902205527983591728
	10^{-6}	$-1.0838219987055633167420 \times 10^{-5}$	1.0903901979676181922027
	10^{-3}	$-1.0830217272699306353823 \times 10^{-2}$	1.0903676084368747014188
	0.1	$-1.0017361246000295077597$	1.0880849795232065695066
	0.2	$-1.8307536307976741340446$	1.0856870428441861495175
	0.3	$-2.4728751913312800023488$	1.0831905688195361120981
	0.4	$-2.9118646357275483221852$	1.0805894594508077531837
	0.5	$-3.1290639840670673370871$	1.0778773551066907014621
	0.6	$-3.1029496606419995148543$	1.0750476366842587490431
	0.7	$-2.8085861582626784271324$	1.0720934388721887733048
	0.8	$-2.2169484754364344826915$	1.0690076796720994475855
	0.9	$-1.2940750966885695874782$	1.0657831136199245401243
	$1-10^{-3}$	$-1.4926710809341307824182 \times 10^{-2}$	1.0624468737640365743670
	$1-10^{-6}$	$-1.4947570254374386527447 \times 10^{-5}$	1.0624124539748200894711
	$1-10^{-9}$	$-1.4947591122372455857985 \times 10^{-8}$	1.0624124195473693182167
	$1-10^{-12}$	$-1.4947591143240462484107 \times 10^{-11}$	1.0624124195129418597827
	$1-10^{-100}$	$-1.4947591143261351379637 \times 10^{-99}$	1.0624124195129073978623

Table 5
Normalized order 70 values of the thermal diffusion related quantities $d_1^{(70)\star}$ and $d_{-1}^{(70)\star}$ for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe as functions of the mixture fractions of the lighter constituents.

Mixture	x_1	$d_1^{(70)\star} \times 10^2$	$d_{-1}^{(70)\star} \times 10^2$
He:Ne	10^{-100}	6.7731889521219033258649	$0.079115585974760126379754 \times 10^{-99}$
	10^{-12}	6.7731889521182510376156	$0.079115585974614355166450 \times 10^{-11}$
	10^{-9}	6.7731889484696150750267	$0.079115585828988913067970 \times 10^{-8}$
	10^{-6}	6.7731852998321243859635	$0.079115440203538368399637 \times 10^{-5}$
	10^{-3}	6.7695351334987884233048	$0.078969806273070044473211 \times 10^{-2}$
	0.1	6.3918794667852847580958	0.064419544318686234501397
	0.2	5.9748995702774248217507	0.098683346138541817412910
	0.3	5.5159904699947163705690	0.10093607633095847446505
	0.4	5.0072057864970334265045	0.068215103919135092887566
	0.5	4.4382924888213206829316	$-0.0038533127133321689529103$

(*continued on next page*)

Table 5 (*continued*)

Mixture	x_1	$d_1^{(70)\star} \times 10^2$	$d_{-1}^{(70)\star} \times 10^2$
	0.6	3.7957716192049345547600	−0.1215338008047818484215 9
	0.7	3.0615327679603599249122	−0.2937401558020335625700 5
	0.8	2.2106112845988171179616	−0.5332573962206772876342 2
	0.9	1.2075312681655995592318	−0.8587754378619050924373 4
	$1-10^{-3}$	$1.3320111715876871332661 \times 10^{-2}$	−1.2933337835576328518242
	$1-10^{-6}$	$1.3334135131141428625369 \times 10^{-5}$	−1.2984167404658051308355
	$1-10^{-9}$	$1.3334149170929273937529 \times 10^{-8}$	−1.2984218312022470878363
	$1-10^{-12}$	$1.3334149184969078175031 \times 10^{-11}$	−1.2984218362929913183199
	$1-10^{-100}$	$1.3334149184983132033143 \times 10^{-99}$	−1.2984218362980871583983
He:Ar	10^{-100}	5.6202118882073375576658	$3.4669461409060272940011 \times 10^{-99}$
	10^{-12}	5.6202118882051655991678	$3.4669461409040412989134 \times 10^{-11}$
	10^{-9}	5.6202118860353790586203	$3.4669461389200322041945 \times 10^{-8}$
	10^{-6}	5.6202097162477443141838	$3.4669441549088329537595 \times 10^{-5}$
	10^{-3}	5.6180388336869041526718	$3.4649580375883119496560 \times 10^{-2}$
	0.1	5.3912826475288084022528	0.0324556126078021954251 36
	0.2	5.1352904666355312524385	0.0594104720902110745165 78
	0.3	4.8454908178896878969324	0.0787871701389684440992 00
	0.4	4.5126379668050286840879	0.0876232537016366914548 55
	0.5	4.1236280252374611370588	0.0815570522949833714670 83
	0.6	3.6591272142184377618741	0.0539124872090726182318 47
	0.7	3.0891502312734682439763	−0.0060377404993051326650322
	0.8	2.3641199688434479275807	−0.1166451328584840632927 8
	0.9	1.3947744159988341475267	−0.3120503643659910352601 1
	$1-10^{-3}$	$1.7112253513407576115791 \times 10^{-2}$	−0.6590026335391774527483 6
	$1-10^{-6}$	$1.7152391460212866431869 \times 10^{-5}$	−0.6637138327256909251675 8
	$1-10^{-9}$	$1.7152431702020220859250 \times 10^{-8}$	−0.6637185593640148822971 0
	$1-10^{-12}$	$1.7152431742262132351023 \times 10^{-11}$	−0.6637185640906686878786 1
	$1-10^{-100}$	$1.7152431742302414544813 \times 10^{-99}$	−0.6637185640954000730848 8
He:Kr	10^{-100}	4.2200063148834996058684	$1.2383598580963337931989 \times 10^{-99}$
	10^{-12}	4.2200063148820641913475	$1.2383598580962723014680 \times 10^{-11}$
	10^{-9}	4.2200063134480850842319	$1.2383598580348420616961 \times 10^{-8}$
	10^{-6}	4.2200048794682403971605	$1.2383597966039907608708 \times 10^{-5}$
	10^{-3}	4.2185701615167994620797	$1.2382977535763354228389 \times 10^{-2}$
	0.1	4.0685074743197002029314	0.0122537575375459633527 30
	0.2	3.8984300901231444351442	0.0239047090734000020114 22
	0.3	3.7047126789561378877330	0.0342246848453189222390 37
	0.4	3.4802612633397051099485	0.0420515087239622174401 18
	0.5	3.2147204558103717283401	0.0454751515926400112008 30
	0.6	2.8922066101814184892296	0.0412312205813143638821 91
	0.7	2.4867737197312108955953	0.0234349123140045550111 24
	0.8	1.9523371575492170270755	−0.0193445187086994743187 31
	0.9	1.1968073272560080237081	−0.1118646036722760384734 8
	$1-10^{-3}$	$1.5644185550670944681238 \times 10^{-2}$	−0.3146355183145986128308 6
	$1-10^{-6}$	$1.5694328017042747304561 \times 10^{-5}$	−0.3176995991516618792341 1
	$1-10^{-9}$	$1.5694378337668949736862 \times 10^{-8}$	−0.3177026774147966926253 2
	$1-10^{-12}$	$1.5694378387989754752915 \times 10^{-11}$	−0.3177026804930740649670 0
	$1-10^{-100}$	$1.5694378388040125929286 \times 10^{-99}$	−0.3177026804961554237123 3
He:Xe	10^{-100}	3.4673186962665970744656	$0.5937629666465030834817 9 \times 10^{-99}$
	10^{-12}	3.4673186962656295647936	$0.5937629666466471813509 1 \times 10^{-11}$
	10^{-9}	3.4673186952990874019362	$0.5937629667906009524482 2 \times 10^{-8}$
	10^{-6}	3.4673177287563621145681	$0.5937631107442226884220 9 \times 10^{-5}$
	10^{-3}	3.4663506231837592181431	$0.5939069147470213359841 9 \times 10^{-2}$
	0.1	3.3644874076394762909512	0.0060638731601952336791200
	0.2	3.2473820810559992356527	0.0122791066128538259572 60
	0.3	3.1119162390641822820440	0.0183966967774790961874 57
	0.4	2.9522233048447136950036	0.0239521635747361065169 57
	0.5	2.7594972740168623722644	0.0280706923531464205213 50
	0.6	2.5197567768990965439503	0.0290537406631204965365 88
	0.7	2.2091059232089267519782	0.0234222416625769292776 81
	0.8	1.7823244353264621330658	0.0034200960048293677801596
	0.9	1.1398682451462442845056	−0.0507481920410073518283 15
	$1-10^{-3}$	$1.6092867801281210588153 \times 10^{-2}$	−2.0042654034354475370831
	$1-10^{-6}$	$1.6162530382470888970178 \times 10^{-5}$	−2.0297275153200616597052
	$1-10^{-9}$	$1.6162600377374446302420 \times 10^{-8}$	−2.0297531370138195988555
	$1-10^{-12}$	$1.6162600447369683821691 \times 10^{-11}$	−2.0297531626356737874865
	$1-10^{-100}$	$1.6162600447439749124847 \times 10^{-99}$	−2.0297531626613212893374
Ne:Ar	10^{-100}	6.8051361546355200044767	$-0.0418790222549856695447 61 \times 10^{-99}$
	10^{-12}	6.8051361546325033885029	$-0.0418790222551272791431 10 \times 10^{-11}$
	10^{-9}	6.8051361516189040287427	$-0.0418790223965952679568 71 \times 10^{-8}$
	10^{-6}	6.8051331380175448445693	$-0.0418791638646467312594 84 \times 10^{-5}$
	10^{-3}	6.8021175363202510788528	$-0.0420206945971337964441 43 \times 10^{-2}$
	0.1	6.4824989441155266206228	−0.0567002429443062184013 78

Table 5 (*continued*)

Mixture	x_1	$d_1^{(70)\star} \times 10^2$	$d_{-1}^{(70)\star} \times 10^2$
	0.2	6.11365110178787878 61433	−0.14597822768854675958116
	0.3	5.69111187889347941 62800	−0.27301468281165250150124
	0.4	5.20559033300139217 09977	−0.44428434727904906823195
	0.5	4.64539644137570672 00473	−0.66798405442534408636952
	0.6	3.99560329647532339 98547	−0.95464412580444685801376
	0.7	3.23682718247424104 12939	−1.31802210712445639 09314
	0.8	2.34340354651546092 00701	−1.77644436204407089 51122
	0.9	1.28057511034012879 98939	−2.35488246965867063 58720
	$1-10^{-3}$	1.40888469378659796 04187 $\times 10^{-2}$	−3.08003329323923953 81407
	$1-10^{-6}$	1.41030401502354322 86031 $\times 10^{-5}$	−3.08827513964890535 73493
	$1-10^{-9}$	1.41030543571322010 46204 $\times 10^{-8}$	−3.08828339164661725 23173
	$1-10^{-12}$	1.41030543713391115 12870 $\times 10^{-11}$	−3.08828339989862512 54943
	$1-10^{-100}$	1.41030543713533326 44482 $\times 10^{-99}$	−3.08828339990688539 36457
Ne:Kr	10^{-100}	6.98420207654390739 50255	0.06236586223741389964 3744 $\times 10^{-99}$
	10^{-12}	6.98420207654133758 46353	0.06236586223733207564 5615 $\times 10^{-11}$
	10^{-9}	6.98420207397409700 31577	0.06236586215558990147 0987 $\times 10^{-8}$
	10^{-6}	6.98419950673185581 57922	0.06236578041337153349 4848 $\times 10^{-5}$
	10^{-3}	6.98163060379787788 01498	0.06228399397042146390 5658 $\times 10^{-2}$
	0.1	6.70955764929824437 16883	0.05370765223636239108 0685
	0.2	6.39477997794582242 55944	0.08788772551622598454 6447
	0.3	6.03096542295171218 93296	0.09835992098471353469 4866
	0.4	5.60616650964349192 21748	0.07937825555404844018 1948
	0.5	5.10393292687073779 00418	0.02285348375392360051 6486
	0.6	4.50090374874992758 40346	−0.08294486284922012203 3484
	0.7	3.76263139575367954 48026	−0.25566505926163533829 738
	0.8	2.83588478389161184 26785	−0.52311569402626402459 159
	0.9	1.63334144885001473 16851	−0.93179482271187317896 422
	$1-10^{-3}$	1.92722726902705904 36144 $\times 10^{-2}$	−1.55790172292221133 02483
	$1-10^{-6}$	1.93076305191260443 96654 $\times 10^{-5}$	−1.56579212992576559 97544
	$1-10^{-9}$	1.93076659452706640 23636 $\times 10^{-8}$	−1.56580003921298350 50973
	$1-10^{-12}$	1.93076659806968770 94628 $\times 10^{-11}$	−1.56580004712228964 11730
	$1-10^{-100}$	1.93076659807323387 69442 $\times 10^{-99}$	−1.56580004713020686 45514
Ne:Xe	10^{-100}	6.39581666847933236 88788	0.04719410798011357641 2401 $\times 10^{-99}$
	10^{-12}	6.39581666847732215 60625	0.04719410798008443845 5537 $\times 10^{-11}$
	10^{-9}	6.39581666646911955 13166	0.04719410795097561951 9709 $\times 10^{-8}$
	10^{-6}	6.39581465826523228 76147	0.04719407884212893168 4690 $\times 10^{-5}$
	10^{-3}	6.39380517099558330 39664	0.04716494222125251168 3547 $\times 10^{-2}$
	0.1	6.18102758409885649 33328	0.04397849856556169556 9163
	0.2	5.93437757838201487 93993	0.08009191554404616356 8457
	0.3	5.64764031243918313 43909	0.10552906614673417712 796
	0.4	5.30933130095615603 26037	0.11621466919795972869 582
	0.5	4.90285199774868968 54760	0.10603867123828231923 456
	0.6	4.40316269225412170 59131	0.06544624479241779133 7443
	0.7	3.77038283895224362 13371	−0.02133027817582925007 8049
	0.8	2.93631791658718177 04242	−0.18221900361363785992 687
	0.9	1.77246569527577445 04112	−0.47171982019852051847 597
	$1-10^{-3}$	2.24659193486906536 12256 $\times 10^{-2}$	−1.00483424984251985 38760
	$1-10^{-6}$	2.25278110289735982 95375 $\times 10^{-5}$	−1.01225018205002178 26111
	$1-10^{-9}$	2.25278731037507323 77271 $\times 10^{-8}$	−1.01225762472879580 14642
	$1-10^{-12}$	2.25278731658256931 65437 $\times 10^{-11}$	−1.01225763217150140 5785
	$1-10^{-100}$	2.25278731658878302 63507 $\times 10^{-99}$	−1.01225763217895156 1572
Ar:Kr	10^{-100}	6.88643311233593074 96360	−0.02805775170474052419 8031 $\times 10^{-99}$
	10^{-12}	6.88643311233216238 37286	−0.02805775170495069166 6228 $\times 10^{-11}$
	10^{-9}	6.88643310856756484 00445	−0.02805775191490799241 6812 $\times 10^{-8}$
	10^{-6}	6.88642934396781602 16837	−0.02805796187223079382 6653 $\times 10^{-5}$
	10^{-3}	6.88266253857724408 95402	−0.02826794127024460539 3947 $\times 10^{-2}$
	0.1	6.48698754158649286 02251	−0.04932012019867955075 5700
	0.2	6.03994500664542035 99592	−0.14235547359008088927 793
	0.3	5.54114913756225585 41078	−0.28139580139606622447 599
	0.4	4.98554188721070649 71994	−0.46948314036131063177 871
	0.5	4.36697807845259895 03508	−0.71054274923841860333 431
	0.6	3.67798131481057873 55294	−1.00956032095634017 32183
	0.7	2.90942057852664624 77925	−1.37282111339552588 76623
	0.8	2.05007757136460460 10223	−1.80823382825060040 27788
	0.9	1.08606074438521409 08906	−2.32577299504636621 09105
	$1-10^{-3}$	1.15323639024081931 04251 $\times 10^{-2}$	−2.93144859259081389 48735
	$1-10^{-6}$	1.15395280644209314 08134 $\times 10^{-5}$	−2.93808396436121461 43970
	$1-10^{-9}$	1.15395352326770213 01728 $\times 10^{-8}$	−2.93809060525320378 02664
	$1-10^{-12}$	1.15395352398452814 88232 $\times 10^{-11}$	−2.93809061189410129 26805
	$1-10^{-100}$	1.15395352398524569 23859 $\times 10^{-99}$	−2.93809061190074883 77435
Ar:Xe	10^{-100}	7.11715390729083005 62494	0.06140483684807324698 4621 $\times 10^{-99}$
	10^{-12}	7.11715390728765134 14605	0.06140483684792510097 9350 $\times 10^{-11}$

(*continued on next page*)

Table 5 (*continued*)

Mixture	x_1	$d_1^{(70)\star} \times 10^2$	$d_{-1}^{(70)\star} \times 10^2$
	10^{-9}	7.117153904112115 2653678	$0.06140483669992724 1674274 \times 10^{-8}$
	10^{-6}	7.117150728574084 9409840	$0.06140468870202849 5058243 \times 10^{-5}$
	10^{-3}	7.113973235429225 1822722	$0.06125665132875737 9602117 \times 10^{-2}$
	0.1	6.7787841495671010700819	0.0461608022515552335 71530
	0.2	6.3951967203208228140819	0.0598092702923390963 13736
	0.3	5.9587860703505247767649	0.0370979590654581453 39902
	0.4	5.4597818473535817268260	−0.0271835047498160138 92827
	0.5	4.8854414731032027318782	−0.1401024497732912853 2683
	0.6	4.2188382477928903070849	−0.3113333793933076299 2877
	0.7	3.4369859494010128912307	−0.5543294513644011104 9190
	0.8	2.5078304550395923161791	−0.8882156109654580709 1737
	0.9	1.3852136913732217770068	−1.3409778992621535820615
	$1-10^{-3}$	$1.5463803327197854968631 \times 10^{-2}$	−1.9479814271398349369849
	$1-10^{-6}$	$1.5482044672106161501217 \times 10^{-5}$	−1.9551077863482633735572
	$1-10^{-9}$	$1.5482062935722524752971 \times 10^{-8}$	−1.9551149239518924436738
	$1-10^{-12}$	$1.5482062953986163415740 \times 10^{-11}$	−1.9551149310895073310806
	$1-10^{-100}$	$1.5482062954004445336346 \times 10^{-99}$	−1.9551149310966520907390
Kr:Xe	10^{-100}	6.3625549899831895174248	$-0.15782386164498711599790 \times 10^{-99}$
	10^{-12}	6.3625549899793902582758	$-0.15782386164514644035730 \times 10^{-11}$
	10^{-9}	6.3625549861839303665803	$-0.15782386180431147543115 \times 10^{-8}$
	10^{-6}	6.3625511907221944273805	$-0.15782402096938441078490 \times 10^{-5}$
	10^{-3}	6.3587538842579964679683	$-0.15798322391708178446438 \times 10^{-2}$
	0.1	5.9636667836211603015102	−0.17414898516874150763973
	0.2	5.5247078393066594356673	−0.38263570760581984327531
	0.3	5.0421099846124873843107	−0.62828095028057410539187
	0.4	4.5117735202624057340845	−0.91434643679732453115280
	0.5	3.9289686172591868368514	−1.2446166097655892116625
	0.6	3.2882133118674370980733	−1.6234999959438491858183
	0.7	2.5831213417498324688684	−2.0561558346532122883624
	0.8	1.8062107283217263199401	−2.5486536171737218121736
	0.9	0.94866071849567548092291	−3.1081759619179639052944
	$1-10^{-3}$	$0.99767932353274393356936 \times 10^{-2}$	−3.7365219612687759475510
	$1-10^{-6}$	$0.99819569574019978138763 \times 10^{-5}$	−3.7432724554502830258084
	$1-10^{-9}$	$0.99819621233987190871689 \times 10^{-8}$	−3.7432792102218728542465
	$1-10^{-12}$	$0.99819621285647180841529 \times 10^{-11}$	−3.7432792169766487233506
	$1-10^{-100}$	$0.99819621285698892543223 \times 10^{-99}$	−3.7432792169834102607614

Table 6
Parameters at STP† of the gases considered in this work.

Gas	Molecular weight (amu) [23]	Molecular diameter $\times 10^8$ (cm) [1]
He	4.002602	2.193
Ne	20.1797	2.602
Ar	39.948	3.659
Kr	83.798	4.199
Xe	131.293	4.939

† STP signifies a temperature of 0 °C and a pressure of 1 atm.

Table 7
Limiting values of the normalized thermal conductivity coefficients, $[\lambda]^{\star}_{70,\ \mathrm{Wynn}}$, for binary mixtures of the noble gases He, Ne, Ar, Kr, Xe as functions of the mixture fractions of the lighter constituents obtained by applying the *Mathematica*® function `SequenceLimit` to the sequences of values from each of the first 70 orders of the Sonine polynomial expansion employed.

x_1	He:Ne	He:Ar
10^{-100}	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080
10^{-12}	1.0252181683234992998804708809 40	1.02521816832365617160484992
10^{-9}	1.02521816837043692023458013 75	1.02521816852730864427479
10^{-6}	1.0252182153080707225394572 8	1.02521837217945593827 70
10^{-3}	1.0252651662654557016332385	1.0254216991946380910 83
0.1	1.0299336122208409889383 63	1.0425705726479693427
0.2	1.0342711214919028713242832	1.05461121996223147659
0.3	1.0377441997218291839800727	1.06221869388672293670 2
0.4	1.0400309786140577438458422 0	1.06601506457572359017 41
0.5	1.0409508018113008633496580 61	1.06643530478454629852 699
0.6	1.0404352117351041030692180 291	1.06377177329386476510 6500
0.7	1.0385000964002621604528579 6666	1.05820570498081538108 04488
0.8	1.0352239452661138543494102 83007	1.04983907309052831842 66099
0.9	1.0307359485698434831137645 6581078	1.03875651300052571585 4592297
$1-10^{-3}$	1.0252776790844339602331887 71656782747	1.02536339165456079106 9261181841521
$1-10^{-6}$	1.0252182278733048580961182 55809071649	1.02521831362331773776 5759496046063828
$1-10^{-9}$	1.0252181683830022068588382 06800993458	1.02521816846875225691 070124977657560893
$1-10^{-12}$	1.0252181683235118651671476 2778430180623	1.02521816832359761521 723666274892113525
$1-10^{-100}$	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080

Table 7 (*continued*)

x_1	He:Kr	He:Xe
10^{-100}	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080
10^{-12}	1.02521816832387842029535	1.025218168324098341179
10^{-9}	1.025218168749557333842	1.025218168969478216
10^{-6}	1.025218594427210104	1.02521881434660117
10^{-3}	1.02564301420239973	1.0258614492348593
0.1	1.0575547191488317	1.069718856501851
0.2	1.07567732424800302	1.090946448927887
0.3	1.085065924232252734	1.1007030665750814
0.4	1.088431043922897363	1.1035362533662975
0.5	1.087188691594638633	1.10142618839514720
0.6	1.08205916244733634099	1.095192857402818518
0.7	1.073335564521470226453	1.0849874866969934562
0.8	1.0610194420757869349593	1.0704532926815110709
0.9	1.04494760814120670234615	1.050781014791557230636
$1-10^{-3}$	1.025429633162470768135407181	1.02549933413931509560167506
$1-10^{-6}$	1.02521837987723151956140390189689	1.0252184496491771302082875061
$1-10^{-9}$	1.0252181685350061825403934983961960	1.025218168604778198062515204861
$1-10^{-12}$	1.025218168323663869142878202332480786	1.02521816832373364115847023444575273
$1-10^{-100}$	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080
x_1	Ne:Ar	Ne:Kr
10^{-100}	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080
10^{-12}	1.02521816832347004892315427010120458970	1.025218168323523547392339438998
10^{-9}	1.025218168341185962891029390103396	1.0252181683946844320333305207
10^{-6}	1.0252181860570864392008946665369619	1.025218239555512713713449248
10^{-3}	1.02523588846206856315880263087782	1.02528934399912160083787507
0.1	1.0268525698676256741496602520056	1.0317538034466545185316162
0.2	1.0281934714488915183683438286106	1.03703457196302089223279512
0.3	1.02921738020793234181956907354269	1.04096603112360821703647572
0.4	1.029900081229369943276682339282927	1.043476855355736301929171
0.5	1.030215949156119081346441326073007	1.044507169432041590784811356
0.6	1.030137261963999160207270661128510	1.0439985955546553411655331714
0.7	1.0296335518883510773651644338145078	1.04188656854128634230706704345
0.8	1.02867108161706677529731721407818457	1.0380969495951095201363274227354
0.9	1.02721264826807272602467866135220343	1.032553557996932673245215478638602
$1-10^{-3}$	1.0252409015804517672381685310852867758	1.02530021186447871442519071043054617643
$1-10^{-6}$	1.0252181910855254369764296644096108038	1.02521825045157134245835927428867355946
$1-10^{-9}$	1.0252181683462144172200007537833810390	1.02521816840558051883188160515601329246
$1-10^{-12}$	1.02521816832347507737749857267129285887	1.02521816832353444347916615986588178461
$1-10^{-100}$	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080
x_1	Ne:Xe	Ar:Kr
10^{-100}	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080
10^{-12}	1.0252181683236047194183992965	1.0252181683234604898878520487048307634
10^{-9}	1.02521816847585645794287594	1.0252181683316269276043398068280595
10^{-6}	1.0252183207273883113155541	1.025218176498066684313818961845019
10^{-3}	1.02537036571341900031626	1.02522634496352952470958983472354
0.1	1.0385045279376701883941	1.026041972182759490707888549975
0.2	1.04826640158926140711657	1.026825787427664537509867678628
0.3	1.05493898195335992241080	1.0274987585311430617256149396016
0.4	1.058807426601329051621293	1.02800063601515905754262048516934
0.5	1.060035867583680514869077	1.028281222566677577947753161795400
0.6	1.05868150110502293201057481	1.0282996460522037136913352605455207
0.7	1.054698447614547763530683129	1.0280235657720448767911156993876201
0.8	1.047937993669552804899377751	1.027428417724990778299321259240671492
0.9	1.03816797675763872317592017410	1.026496807098092139301042953855765482
$1-10^{-3}$	1.025362762347695018467799084958151203	1.0252326904595083265683349253177422434
$1-10^{-6}$	1.0252183130551717891830055836918280423	1.02521818286313630178105785597594494536
$1-10^{-9}$	1.02521816846818417218901301890637760402	1.02521816833799201680998481789246269899
$1-10^{-12}$	1.02521816832359704713257620147825784345	1.02521816832346685497707728190820785950
$1-10^{-100}$	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080
x_1	Ar:Xe	Kr:Xe
10^{-100}	1.02521816832345231527552527441069478080	1.02521816832345231527552527441069478080
10^{-12}	1.02521816832348692620494043777517	1.0252181683234577755530802668810185826
10^{-9}	1.025218168358063244681772981865	1.0252181683289125928269119215716213766
10^{-6}	1.02521820293437280582300208881	1.0252181737837262608368657364551519212
10^{-3}	1.02525277028506053372446985041	1.02522362498942909609132319308034432
0.1	1.028548878021145605382296325	1.0257259902623583823512265551450869
0.2	1.031470814648072957453159551	1.0261492215722242213005229749095566
0.3	1.0337974098516747615261165571	1.02647616377549971563271556296998718
0.4	1.03538502096898526302270981041	1.02669582320929315566985027582703316
0.5	1.03612646824127711187251486888	1.02679784277496595713255591531988119
0.6	1.035944324866904389858333631599	1.026772437570126674141014390632986689
0.7	1.0347850603709656719589487065358	1.026610338499758148153216018067542602

(*continued on next page*)

Table 7 (*continued*)

x_1	Ar:Xe	Kr:Xe
0.8	1.03261523758285882681906185711368	1.026302749083486114139961655010977551
0.9	1.02942154853232890433268624770427 39	1.025841322454280378637374969541712161 0
$1-10^{-3}$	1.0252650502799158537646309151525700 1157	1.0252225225121974680937067115206926966 75
$1-10^{-6}$	1.025218215253056200288049882107164545 91	1.0252181753886992495056356565557644633
$1-10^{-9}$	1.025218168370381966791800269268894610 62	1.0252181683305175706593512688929871061
$1-10^{-12}$	1.025218168323499244927089180651824614 65	1.0252181683234593805309175500022449399 9
$1-10^{-100}$	1.025218168323452315275525274410694780 80	1.0252181683234523152755252744106947808 0

Table 8
Limiting values of the normalized diffusion coefficients, $[D_{12}]^{\star}_{70,\ \mathrm{Wynn}}$, for binary mixtures of the noble gases He, Ne, Ar, Kr, Xe as functions of the mixture fractions of the lighter constituents obtained by applying the *Mathematica*® function `SequenceLimit` to the sequences of values from each of the first 70 orders of the Sonine polynomial expansion employed.

x_1	He:Ne	He:Ar
10^{-100}	1.0183168264812160978478 6	1.0276991547067152383
10^{-12}	1.0183168264811956098342 6	1.0276991547066927650
10^{-9}	1.0183168264607280842461 3	1.0276991546842419330
10^{-6}	1.0183168059932038223074 3	1.0276991322334054092
10^{-3}	1.0182963397947762978958 5	1.0276766769121204747
0.1	1.016281021072661370258670	1.02540499352297683991
0.2	1.0142707324702496526939133	1.02300920258291654111 6
0.3	1.0122868360742158519349962 2	1.0204996880937536582 57
0.4	1.0103326982688267478352709 29	1.0178647622219440512661
0.5	1.0084148710655480589782700635	1.0150946599921391752068 3
0.6	1.0065441532326563140594791630 1	1.0121848343469760904817 46
0.7	1.0047371827722631309030849675 07	1.0091430858762757264156966
0.8	1.0030188170227353370556218820 082	1.0060053800340151210504240 7
0.9	1.0014256873072236051839379413 5887	1.0028725699954631378588986255
$1-10^{-3}$	1.0000244973007121061709166740112 84060	1.0000267165334709955682543273498
$1-10^{-6}$	1.0000114859182961044521136949612 643382	1.0000009119184908804051529932528 69
$1-10^{-9}$	1.0000114729196038577973523905875 5497366559	1.0000008861553395349889251776593 25799
$1-10^{-12}$	1.0000114729066051782563190265019 10294670773	1.0000008861295764252605259753453 7288044
$1-10^{-100}$	1.0000114729065921665650994792401 32121638603	1.0000008861295506363619406079880 8437619

x_1	He:Kr	He:Xe
10^{-100}	1.0352327170099902	1.038293259238067
10^{-12}	1.0352327170099647	1.038293259238044
10^{-9}	1.035232716984525	1.038293259215191
10^{-6}	1.0352326915449476	1.03829323636272
10^{-3}	1.035207245709673	1.038270375794141
0.1	1.032619726507292	1.035919533616143
0.2	1.02985741037526330	1.033350957713110
0.3	1.026919155610518715	1.0305484178041635
0.4	1.0237753359212331549	1.02746380710207373
0.5	1.0203937341131676153	1.02403760817321869
0.6	1.016742003941597094644	1.020196992910507289
0.7	1.01279563217104718735 8	1.0158577112681115963
0.8	1.008561427105538779567 3	1.0109426505517480600
0.9	1.004148386736273702520 06	1.00547186749256551626 6
$1-10^{-3}$	1.000036748907021914294188059	1.000048631385523874229652 9
$1-10^{-6}$	1.000000086478966164652556705109	1.000000056997742672121923162 5
$1-10^{-9}$	1.000000049895904942812999125707 69	1.000000008550924105006657295 57
$1-10^{-12}$	1.000000049859321961375773642613 3272	1.000000008502477414992916557 5420
$1-10^{-100}$	1.000000049859285341774815243844 4619	1.000000008502428919807846301 0392

x_1	Ne:Ar	Ne:Kr
10^{-100}	1.0062522721826473052154086523687	1.01550568657443872101614 6
10^{-12}	1.0062522721826426354630508485802	1.01550568657442691026671 73
10^{-9}	1.0062522721779775528564358261868	1.01550568656262797158378 07
10^{-6}	1.0062522675128937772047084606227 7	1.01550567476368604087779 36
10^{-3}	1.0062476012598596337457221475238 3	1.01549387257307823222985 28
0.1	1.0057733721101209322929328576277 5	1.01429117813796819697770 48
0.2	1.0052697314261031975369867953890 8	1.01300601350130224961773 321
0.3	1.0047400410478414543051806634490 64	1.01164440680785693696580 514
0.4	1.0041830960307227429356090596206 82	1.01020077707304471929019 25180
0.5	1.0035979230487959657036251566724 74	1.00867052440755816418260 85930
0.6	1.0029839853027986999440353112783 66	1.00705155999436758391681 18215
0.7	1.0023415123293584781196398996357 6840	1.00534733189709804488189 3037255
0.8	1.0016720343864484974124685814946 30662	1.00357292436296505681743 01785987
0.9	1.0009792579461158030780948154969 897966	1.00176774949029597287817 2662105549
$1-10^{-3}$	1.0002776505333489962426871183921 656793087	1.00003969457203763023007 3255900652718 01
$1-10^{-6}$	1.0002705286485104167135222468484 0589341298	1.00002309295705047192764 7553569757522954
$1-10^{-9}$	1.0002705215264551998259389613668 3784476896359	1.00002307636769105096409 9530582121762616 60
$1-10^{-12}$	1.0002705215193331444415416822757 9680690715288	1.00002307635110170384585 1287115961922233499
$1-10^{-100}$	1.0002705215193260152569725483557 9515866261690	1.00002307635108509789279 2282642874938329948

Table 8 (*continued*)

x_1	Ne:Xe	Ar:Kr
10^{-100}	1.021992270374199610512	1.0068324667203317164537282110923
10^{-12}	1.021992270374185128839	1.0068324667203251605675533016056
10^{-9}	1.02199227035971793708	1.0068324667137758302784702472518
10^{-6}	1.021992255892520901500	1.0068324601644451927187955202597
10^{-3}	1.021977783414322344989	1.0068259104855023554461421208507
0.1	1.0204890430707023341454	1.00617356742366568743679879765311
0.2	1.01886613675375711528625	1.00550881496124196295677072971303
0.3	1.01710757344347706810780	1.004839524242155165457467110883483
0.4	1.015195167849895726846184	1.004167291669056657458540126045356
0.5	1.0131089292344308255887553	1.0034940900571199643452001401556915
0.6	1.0108286726193813454896806	1.00282238574888487655572579088052585
0.7	1.008338740357427117273032259	1.00215528563176738427247865729331590
0.8	1.0056409566784892847078258504	1.001496724076594829779736134376633777
0.9	1.002790746805246591579925273898	1.00085170289122998366340371750084555853
$1-10^{-3}$	1.00003040719651646667805835366715134	1.000232728674352654341060809980374440233
$1-10^{-6}$	1.0000045033143773098857599568661710146	1.00022660746498123437495542712683411270398
$1-10^{-9}$	1.000004477443935650895636036801379081496	1.0002266013451633539675371204877387427614731
$1-10^{-12}$	1.000004477418065242860499454790227197878407	1.00022660133904353748014518509949508113488256
$1-10^{-100}$	1.000004477418039346556193636460977064828465	1.00022660133903741153771675775516385998098386

x_1	Ar:Xe	Kr:Xe
10^{-100}	1.01223484970167383677219424	1.0042215316500266071664437767945052
10^{-12}	1.01223484970166278750984580	1.0042215316500232467643361718352947
10^{-9}	1.01223484969062457442219133	1.0042215316466662050584638993587417
10^{-6}	1.01223483865240991801245043	1.0042215282896241242683548020370305
10^{-3}	1.01222379886889776967460649	1.0042181708726392933168578256652282
0.1	1.011114134478969456323504791	1.0038817543497835431662276659373973
0.2	1.0099616503843936792566159600	1.00353458457110902809594400165189914
0.3	1.0087775859371242079125200981	1.00318019315073501914154303723700679
0.4	1.007563075262406250005323804825	1.00281882992486308810499074754102546
0.5	1.006320755541292932953026566509	1.002450845821952800579697546097343598
0.6	1.0050556258810593802641532873928	1.002076720631906145202910082737402712
0.7	1.003776393622580599577078058614114	1.001697098064084212035542127394146072
0.8	1.002497623115601267751291674575991	1.001312830206529645525750737174279678
0.9	1.0012432298262261108163629364210728	1.00092503416692994054791734845180412038
$1-10^{-3}$	1.0000636769289761947250006744868289196	1.00053906775222710541632636038292290024681
$1-10^{-6}$	1.0000522751552412447238033510082675917118	1.000535168474396855869010808452547697882641
$1-10^{-9}$	1.000052263759647887517974282109716834663749	1.0005351645751069970858033432533635642858984
$1-10^{-12}$	1.0000522637482523003543345185006418351409890	1.000535164571207707215345513570109945808477288
$1-10^{-100}$	1.0000522637482408933601829207247205649924320	1.000535164571203804022281980533109938163373981

Table 9
Limiting values of the normalized thermal diffusion coefficients, $[D_T]^{\star}_{70,\ \mathrm{Wynn}}$, for binary mixtures of the noble gases He, Ne, Ar, Kr, Xe as functions of the mixture fractions of the lighter constituents obtained by applying the *Mathematica*® function `SequenceLimit` to the sequences of values from each of the first 70 orders of the Sonine polynomial expansion employed.

x_1	He:Ne	He:Ar
10^{-100}	1.1727641524477615750680	1.243619954924111046
10^{-12}	1.1727641524476594232243	1.243619954923981229
10^{-9}	1.1727641523456097313347	1.243619954794293805
10^{-6}	1.1727640502959042014150	1.243619825106835715
10^{-3}	1.172661986942640152255	1.24349010330504121
0.1	1.16240535092409546437959	1.230275721403607946
0.2	1.151733450258779078165309	1.21612494592888583799
0.3	1.1407145035474531018333009	1.20103005324014952888
0.4	1.1293204418648855917929027	1.184829004479504991591
0.5	1.117529248706176910562749919	1.1673297089742302244567
0.6	1.1053264335987840714281299953	1.14830465721066002919153
0.7	1.09270834717810430552186919012	1.127488770472888639366010
0.8	1.079688431796450137406436318265	1.1045900958372877153005938
0.9	1.0663085295817499537119031119613	1.07934489266614582793961747
$1-10^{-3}$	1.0527967781237656598798134621924725	1.052016360439920344186688592327
$1-10^{-6}$	1.0526595763539228477890814796077799	1.051730438837696051382546211154301
$1-10^{-9}$	1.0526594391469324381230441964468612	1.051730152835598667135568915260216
$1-10^{-12}$	1.0526594390097254425317572145192112	1.0517301525495964895710084979199832
$1-10^{-100}$	1.0526594390095880981918208095451919	1.0517301525493102011048976465738973

x_1	He:Kr	He:Xe
10^{-100}	1.29680196405877	1.3178691333862
10^{-12}	1.296801964058623	1.3178691333860
10^{-9}	1.296801963904337	1.3178691332459
10^{-6}	1.296801809618220	1.3178689931225
10^{-3}	1.29664748223939	1.3177288186014
0.1	1.2809161416620045	1.30329652274027

(*continued on next page*)

Table 9 (*continued*)

x_1	He:Kr	He:Xe
0.2	1.26402097767581563	1.28747601991466
0.3	1.24590062648835273	1.270129607629226
0.4	1.226291891983980966	1.25089914161468
0.5	1.20486665657155927	1.2293107547548959
0.6	1.1812088971226820728	1.20472031166527301
0.7	1.15478693860466367423	1.1762289206590653
0.8	1.124931596578372005971	1.1425582112572240534
0.9	1.090878852322846819885 1	1.10192587943654879158
$1-10^{-3}$	1.0525993069657142893516012	1.0530131859884968176080 9
$1-10^{-6}$	1.05219251549937817872734354 6	1.052475658775180015170604
$1-10^{-9}$	1.052192108553317701993973863 48	1.0524751208933221620502948 3
$1-10^{-12}$	1.0521921081463714877940226731957	1.0524751203554399514190671 5
$1-10^{-100}$	1.0521921081459641342261010779148	1.052475120354901530787451533 19
x_1	Ne:Ar	Ne:Kr
10^{-100}	1.1044242646588806480631250339 14	1.161800743578499574398635
10^{-12}	1.10442426465884983840637602236	1.1618007435784344801684
10^{-9}	1.10442426462807099130370985718 7	1.161800743513405344156502
10^{-6}	1.10442423384921348501767274798 2	1.161800678484242984537320
10^{-3}	1.10439344458438660962810274637 2	1.161735622962522524908284
0.1	1.10123534081488759230617485740 08	1.155015473593815121506510 8
0.2	1.097814257196646566346960156481 7	1.147626272482994744389289 5
0.3	1.094134254652962958601539077865 31	1.139545096697000599070023 4
0.4	1.090164980918693247693766614137 60	1.130668770598788769977495 6
0.5	1.085871938058734063475120909481 64	1.120875885894715737122506156
0.6	1.081215884026707059173859701090421	1.110023581801452687242676633 97
0.7	1.076152234776172648787546886718860 1	1.097945186314759766049070063 28
0.8	1.070630577362153258763134639412773 86	1.084451617104771075175724013747
0.9	1.064594539496125430083881737040787143	1.069344970758665071695219133282 04
$1-10^{-3}$	1.058051732023308874700546384076988486 6	1.052647548994092161853691449705390930
$1-10^{-6}$	1.057982619843121196755368987680539505 9	1.052470071795041580745340339418072555
$1-10^{-9}$	1.057982550698972951831791149303101974 3	1.052469894229782001469282456841852676
$1-10^{-12}$	1.057982550629828771608137749490532093 3	1.052469894052216653867810053118711888
$1-10^{-100}$	1.057982550629759558214488468036793940 3	1.052469894052038910777029778194288725
x_1	Ne:Xe	Ar:Kr
10^{-100}	1.206591368318376992471	1.104062881151317652060940072939
10^{-12}	1.206591368318294888289	1.104062881151281102187118148227
10^{-9}	1.206591368236272810274	1.104062881114767778231843997 42
10^{-6}	1.206591286214157592526	1.104062844601436651599672229690
10^{-3}	1.206509226876900658260	1.104026324096864608327527283293
0.1	1.197988800775016129400	1.100334059863211185728864060327
0.2	1.188514330944370746094 8	1.096449686979424259495619612619
0.3	1.178013772318496650034 42	1.092398578103987715292684416286 7
0.4	1.166295194188795489549 00	1.088170339432098328390295111633 6
0.5	1.153117039500250634668426	1.083755347661942718463651616354892
0.6	1.138172358845126150319480 6	1.079144812971654050894421221860 16
0.7	1.121069263379881981406800 85	1.074330945660971588023472893045 3344
0.8	1.101312415019739045037403540	1.069307260099959529231186819069 7444
0.9	1.078311389988056966029644024 38	1.064069068882886420395318587648 21071
$1-10^{-3}$	1.051827558500421000347322524588 4968	1.058669868611716826985824502982 3834600
$1-10^{-6}$	1.051541113256711973818738672468 0335	1.058614305178970314172190261823 7003407
$1-10^{-9}$	1.051540826630418990274918598241 2299	1.058614249604774395743516700749 9634389
$1-10^{-12}$	1.051540826343792516473418187896 0175	1.058614249549200189062551157029 9147781
$1-10^{-100}$	1.051540826343505603086048499893 2885	1.058614249549144559226022900682 8522345
x_1	Ar:Xe	Kr:Xe
10^{-100}	1.138083889772611009944142 06	1.090390220575406248451517188636 59
10^{-12}	1.138083889772552091641910 01	1.090390220575383640667920635657 79
10^{-9}	1.138083889713692707694255 4	1.090390220552798464850614541722 420
10^{-6}	1.138083830854290925968022 37	1.090390197967618297879840388789 254
10^{-3}	1.138024953611838384421782 11	1.090367608436874806493130017469 69
0.1	1.132006865483474247972855 30	1.088084979523206628117318795183 31
0.2	1.125532129205771134717307 468	1.085687042844186180852544128015 13
0.3	1.118617087638305590690599 1751	1.083190568819536128167891414455 644
0.4	1.111216741109872590155149 7142	1.080589459450807761040963343260 3989
0.5	1.103283588580297480086348 71761	1.077877355106690705096917076408 6505
0.6	1.094768024231321759910818 909698	1.075047636684258750618262632114 48991
0.7	1.085619677595188877212293 2957870	1.072093438872188773935857636387 99538
0.8	1.075790594066163301879011 2097551	1.069007679672099447814970894586 439270
0.9	1.065242083736639387711989 556987402	1.065783113619924540198104545911 9579001
$1-10^{-3}$	1.054075521178631407617748 0402544 08639	1.062446873764036574387439457323 2253460
$1-10^{-6}$	1.053959232063619076516133 5565367 00361	1.062412453974820089491258125955 2910657
$1-10^{-9}$	1.053959115739301430297605 8469546 04125	1.062412419547369318236863811672 1450222
$1-10^{-12}$	1.053959115622977077461454 9429874 05991	1.062412419512941859802858083888 6527486
$1-10^{-100}$	1.053959115622860636667789 9371227 54732	1.062412419512907397882496053370 1580748

$$[D_T]^\star_m \equiv \frac{[D_T]_m}{[D_T]_1} = \frac{x_1 M_1^{-1/2} d_1^{(m)\star} + x_2 M_2^{-1/2} d_{-1}^{(m)\star}}{x_1 M_1^{-1/2} d_1^{(1)\star} + x_2 M_2^{-1/2} d_{-1}^{(1)\star}} = \frac{x_1 M_2^{1/2} d_1^{(m)\star} + x_2 M_1^{1/2} d_{-1}^{(m)\star}}{x_1 M_2^{1/2} d_1^{(1)\star} + x_2 M_1^{1/2} d_{-1}^{(1)\star}}, \tag{210}$$

$$[\lambda]_m = [\lambda]^\star_m[\lambda]_1 = -\frac{5}{4}kn\left(\frac{2kT}{m_0}\right)^{1/2} \times \left(x_1 M_1^{-1/2} a_1^{(m)\star} a_1^{(1)} + x_2 M_2^{-1/2} a_{-1}^{(m)\star} a_{-1}^{(1)}\right), \tag{211}$$

$$[\lambda]^\star_m \equiv \frac{[\lambda]_m}{[\lambda]_1} = \frac{\left(x_1 M_1^{-1/2} a_1^{(m)\star} a_1^{(1)} + x_2 M_2^{-1/2} a_{-1}^{(m)\star} a_{-1}^{(1)}\right)}{\left(x_1 M_1^{-1/2} a_1^{(1)\star} a_1^{(1)} + x_2 M_2^{-1/2} a_{-1}^{(1)\star} a_{-1}^{(1)}\right)} = \frac{\left(x_1 M_2^{1/2} a_1^{(m)\star} a_1^{(1)} + x_2 M_1^{1/2} a_{-1}^{(m)\star} a_{-1}^{(1)}\right)}{\left(x_1 M_2^{1/2} a_1^{(1)} + x_2 M_1^{1/2} a_{-1}^{(1)}\right)}. \tag{212}$$

Here, for the specific case of rigid-sphere molecules, one then uses Eqs. (146)–(148) for the omega integrals needed. We have reported some representative order 70 results for $[\lambda]_1$, $a_1^{(1)}$, $a_{-1}^{(1)}$ (in Table 1), $[\lambda]^\star_{70}$, $a_1^{(70)\star}$, $a_{-1}^{(70)\star}$ (in Table 2), $[D_{12}]_1$, $d_0^{(1)}$, $[D_{12}]^\star_{70}$ (in Table 3), $[D_T]_1$, $[D_T]^\star_{70}$ (in Table 4), and $d_1^{(70)\star}$, $d_{-1}^{(70)\star}$ (in Table 5) for binary mixtures of the noble gases He, Ne, Ar, Kr, and Xe. The molecular mass and diameter values used in computing the results given in Tables 1–5 are summarized in Table 6.

As a result of our obtaining normalized values of the transport coefficients for each order of approximation up to 70, we have for each combination of parameters considered, sequences of values that, at least for the thermal conductivities and the diffusion coefficients, are known to be increasing monotonically to fixed limiting values [1]. As a part of our work, we have applied the *Mathematica*® function `SequenceLimit` to each such sequence of values in order to determine the limiting values associated with our order 70 results. The `SequenceLimit` function extrapolates to the limit using the Wynn epsilon algorithm. The extrapolated results, which we have included in Tables 7–9, are truncated at the number of digits that appear to be common to most of the extrapolations done for each point using Wynn orders from 14 to 30 which is the range of Wynn orders that appears to give the most consistent extrapolated values. It is our assessment that the extrapolated values presented in Tables 7–9 are likely correct to the precision shown excepting, perhaps, only the last digit and should be considered the best rigid-sphere, real gas mixture benchmark values available for the diffusion, thermal diffusion, and thermal conductivity coefficients. Even with the last digit considered to be unreliable, the extrapolated values of Tables 7–9 are more than double the precision of order 70 results and are, effectively, equivalent to about order 150 results. We have verified this against the results for simple gases where results to order 200 were reported [17].

9. Discussion and conclusions

Our purpose in this series of papers has been to explore the use of Sonine polynomial expansions to obtain results free of numerical approximations and errors for the transport coefficients and related Chapman–Enskog functions for simple gases and gas mixtures. In our first paper [17], we explored the case of simple gases. In our second paper we extended our initial work to viscosity in binary, real gas mixtures [18]. Now we have extended our work further to include an exploration of diffusion, thermal diffusion, and thermal conductivity and the related Chapman–Enskog solutions for the same set of binary, real gas mixtures. For specific results, we have focused in this work, as we have in our previous works, on rigid-sphere molecules, since the omega integrals are readily available in a simple analytical form. However, we must emphasize that the expressions that we have obtained are completely general and that results for any potential model can now be readily obtained subject only to omega integral values for the given potential model being available to sufficient precision.

We have reported here Sonine polynomial expansions only up to order 70 but the computational tools and programs that we have constructed apply to expansions of arbitrary order and the only limiting factors that we have encountered involve the speed and memory capacity of the computational resources available to us. Nevertheless, because of the high precision of our computations, we have witnessed excellent convergence in all of our results and, via extrapolation, we have been able to obtain transport coefficient values to a precision of more than 14 significant figures for all of the rigid-sphere gas mixtures that we have considered. Note that for the case of viscosity [18], we were able to compare our numerical results with those of Garcia and Siewert [24]. We have not been able to make similar comparisons here as it appears to us that their computations would need to be recast in a different form for a direct comparison with the present results.

Since even in a binary-gas mixture the parameter space is very large (involving mole fractions, mass ratios, size ratios, temperature, etc.), it is always useful to have techniques or expressions such as we have obtained in the current work that make rapid and precise computations possible. We should note, as an example, that even carrying a numerical precision of 400 digits, the inversion of an order 70 matrix required CPU time on the order of only seconds in *Mathematica*®. It is because we have emphasized the development of a very general set of such computational tools and techniques that our work, particularly our comparisons with the results of Takata et al. [19], have been relatively straightforward and quick to obtain. It is this same generality that has allowed us to extend our work in a straightforward manner to additional real gas mixtures and which will allow us to complete an ongoing and much more substantial parametric study which we expect to present as an additional paper in the near future as well as further future studies using more realistic potential models. The attractiveness of our work is obvious with respect to ternary and multiple gas mixtures as no new basic expressions need to be computed and the computations would be straightforward. Efforts in this direction are currently underway.

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