Irrev, Øving 5

1a) $J_q' = L_{qq}\frac{\partial}{\partial x}\left(\frac{1}{T}\right) + L_{q\phi}\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right)$

$j = L_{\phi q}\frac{\partial}{\partial x}\left(\frac{1}{T}\right) + L_{\phi\phi}\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right)$

1b) Peltier effect: Heat transport due to $\frac{\partial \phi}{\partial x} \neq 0$

$$\Pi := F\left(\frac{J_q'}{j}\right)_{dT=0} = F\frac{L_{q\phi}}{L_{\phi\phi}}$$

1c) Seebeck effect: Electric potential gradient due to a heat flux.

Seebeck coeff $:= F\left(\frac{d\phi}{dT}\right)_{j=0} = -\frac{\Pi}{T}$

2a) $j = 10\,\frac{A}{m^2}$, $\lambda = 5\,\frac{W}{mK}$, $T = 300\,K$, $S^*_{c,Pb} = 5\,\frac{J}{molK}$

(I) $J_q' = L_{qq}\frac{\partial}{\partial x}\left(\frac{1}{T}\right) + L_{q\phi}\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right)$

(II) $j = L_{\phi q}\frac{\partial}{\partial x}\left(\frac{1}{T}\right) + L_{\phi\phi}\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right)$

ohm: $K = \frac{L_{\phi\phi}}{T}$

Fourier: $\lambda = \frac{L_{qq}}{T^2}$

$\Pi := F\frac{L_{q\phi}}{L_{\phi\phi}}$

$\frac{L_{q\phi}}{T} = \frac{L_{\phi\phi}}{T}\frac{\Pi}{F} = \frac{K\Pi}{F}$

$J_q' = 0 \rightarrow \frac{L_{q\phi}}{T}\frac{\partial \phi}{\partial x} = -\frac{L_{qq}}{T^2}\frac{\partial T}{\partial x}$

$\frac{\partial \phi}{\partial x} = -\frac{L_{qq}}{L_{q\phi}}\frac{1}{T}\frac{dT}{dx}$

$j_{dT=0} = -\frac{L_{\phi\phi}}{T}\frac{\partial \phi}{\partial x} \rightarrow \frac{\partial \phi}{\partial x} = -\frac{T}{L_{\phi\phi}} j = -\frac{L_{qq}}{L_{q\phi}}\frac{1}{T}\frac{dT}{dx}$

$\frac{dT}{dx} = T^2\frac{L_{q\phi}}{L_{\phi\phi}}\frac{1}{L_{qq}} j = \frac{\Pi}{F\lambda} j = \frac{TS^*_{c}}{F\lambda} j$

$$\frac{dT}{dx} = 3{,}1\cdot 10^{-2}\ \frac{K}{m}$$

2b) $J_q' = L_{qq}\frac{\partial}{\partial x}\left(\frac{1}{T}\right) + L_{q\phi}\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right)$

$j = L_{\phi q}\frac{\partial}{\partial x}\left(\frac{1}{T}\right) + L_{\phi\phi}\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right)$

$j = 0 \rightarrow \frac{L_{\phi\phi}}{T}\frac{\partial \phi}{\partial x} = -\frac{L_{\phi q}}{T^2}\frac{\partial T}{\partial x} \rightarrow \frac{\partial T}{\partial x} = \frac{T L_{\phi\phi}}{L_{\phi q}}\frac{\partial \phi}{\partial x}$

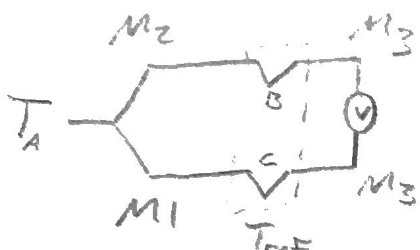
$J'_{q, d\phi=0} = -\frac{L_{qq}}{T^2}\frac{\partial T}{\partial x}$

$\frac{\partial T}{\partial x} = -\frac{T^2}{L_{qq}} J_q' = \frac{T L_{\phi\phi}}{L_{\phi q}}\frac{\partial \phi}{\partial x} = \frac{T F}{\pi}\frac{\partial \phi}{\partial x}$

$\frac{\partial \phi}{\partial x} = -\frac{\pi}{\lambda T F} J_q' = -\frac{S_{e^-}^*}{\lambda F} J_q'$

$\frac{\partial \phi}{\partial x} = -10^{-4}\,\frac{V}{m}$

3a)



$j = 0 \rightarrow \frac{\partial \phi}{\partial x} = -\frac{L_{\phi q}}{L_{\phi\phi}}\left(\frac{1}{T}\right)\frac{\partial T}{\partial x}$

$\int_A^B \frac{\partial \phi}{\partial x}dx = -\pi_2 \int_A^B \left(\frac{1}{T}\right)\frac{\partial T}{\partial x}dx = -\pi_2 \int_A^B \frac{\partial}{\partial x}\ln T\,dx = -\pi_2 \ln\frac{T_B}{T_A}$

$\Delta_{AB}\phi = -\pi_2 \ln\left(\frac{T_B}{T_A}\right)$

$\Delta_{AC}\phi = -\pi_1 \ln\left(\frac{T_C}{T_A}\right)$

$\Delta_{BC}\phi = \Delta_{AC}\phi - \Delta_{AB}\phi = \ln\left(\frac{T_B}{T_A}\right)^{\pi_2} + \ln\left(\frac{T_C}{T_A}\right)^{-\pi_1}$

$\Delta_{BC}\phi = \ln\left(\frac{T_{ref}}{T_A}\right)^{\pi_2 - \pi_1}$

$T_A = T_{ref}\exp\left(\frac{\Delta_{BC}\phi}{\pi_1 - \pi_2}\right)$

Oppg. 3b)

$$\frac{d\Phi}{dT} = 3{,}82 \cdot 10^{-3} \frac{V}{K}$$

$$\Delta\Phi = \frac{d\Phi}{dT} \Delta T = 1{,}15V$$

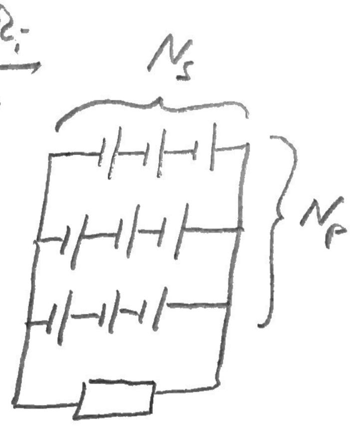
3c) $R = 1{,}8\Omega$ $\quad I_T = 0{,}1$ $\quad \Delta\Phi_i = 1{,}15V$ $\quad R_p = \left(\sum_i \frac{1}{R_i}\right)^{-1} = \frac{R}{N}$

$$R_s = \sum R_i = NR$$

$P = I\Delta\Phi$

$\Delta\Phi = N_s \Delta\Phi_i$

$$I = \frac{\Delta\Phi}{R}, \quad R = \left(\sum_{i=1}^{N_p} \frac{1}{N_s R_i}\right)^{-1} = \frac{N_s R_i}{N_p}$$



$$I = \frac{N_p}{N_s R_i} N_s \Delta\Phi_i = \frac{N_p}{R_i} \Delta\Phi_i$$

$$N_p = \frac{I R_i}{\Delta\Phi_i} = 0{,}16$$

$\underline{N_p = 1 \rightarrow I = 0{,}63A}$

$$N_s = \frac{P}{I\Delta\Phi_i} = 8{,}1$$

$\underline{N_s = 9}$

9 modules in series gives 0,63 A and 6,5 W

3b) Pass current through a thermoelectric module to develop a temperature gradient and transport heat out of the water

$$\left(J_q'\right)_{dT=0} = -\frac{L_{q\phi}}{T}\frac{d\Phi}{dx}, \quad \left(\frac{d\Phi}{dx}\right)_{dT=0} = -\frac{T}{L_{\phi\phi}} j$$

$$J_q' = \frac{L_{q\phi}}{L_{\phi\phi}} j = \frac{\pi}{F} j = \frac{T S_{c}^*}{F} j$$

$\underline{J_q' = 4{,}44 \cdot 10^{-2}\ J/s}$

$Q = 25g \cdot 4{,}18 \frac{J}{g \cdot K} \cdot 5K$

$\underline{Q = 522J}$

$$t = \frac{Q}{J_q'} = 11760s = 3{,}3h$$