Irrev. Øving 2

Opg. 1a) J_i = Massefluks av komponent i

1b) r = reaksjonshastighet, $[r] = \frac{mol}{m^3 s}$

1c) $\frac{\partial}{\partial x}\left(\frac{1}{T}\right)$, gradient i den inverse temperaturen

1d) $\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right)$, $-\frac{\partial \phi}{\partial x}$ = elektrisk felt

Opg. 2a)

$$\sigma = J_q' \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \sum_j J_i\left(-\frac{1}{T}\frac{\partial \mu_{j,T}}{\partial x}\right) + j\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right) + r\left(-\frac{\Delta G}{T}\right)$$

$$\frac{dT}{dt} = 0 \nRightarrow \frac{dT}{dx} = 0$$

$$j = F\sum z_i J_i$$

$$\sigma = J_q' \frac{\partial}{\partial x}\left(\frac{1}{T}\right) - \frac{J_i}{T}\left(\frac{\partial \mu_{i,T}}{\partial x} + F z_i \frac{\partial \phi}{\partial x}\right) + r\left(-\frac{\Delta_r G}{T}\right)$$

2b)

$$\sigma = J_q \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \sum J_i\left(-\frac{1}{T}\frac{\partial \mu_i}{\partial x}\right) + j\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right) + r\left(-\frac{\Delta_r G}{T}\right)$$

$$r = 0, \quad J_i = 0 \ \forall j$$

$$\underline{\sigma = J_q \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + j\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right)}$$

2c) Ficks lov: $J_i = -D\frac{\partial c_i}{\partial x}$, Fourier: $Q = -k\frac{dT}{dx}$

Ohm: $i = -\frac{1}{R}\frac{d\phi}{dx}$, $[R] = \Omega$

$$\vec{J} = \underline{L}\vec{X}, \quad \vec{J} = \begin{pmatrix} J_q' \\ J \\ j \end{pmatrix}, \quad \vec{X} = \begin{pmatrix} \frac{\partial}{\partial x}\left(\frac{1}{T}\right) \\ -\frac{1}{T}\frac{\partial \mu}{\partial x} \\ -\frac{1}{T}\frac{\partial \phi}{\partial x} \end{pmatrix}$$

Antar at alle $L_{ij} = 0$, $j \neq i$

$$J_q' = L_{qq}\frac{\partial}{\partial x}\left(\frac{1}{T}\right) = L_{qq}\left(\frac{-1}{T^2}\frac{\partial T}{\partial x}\right), \quad L_{qq} = kT^2 \rightarrow \text{Fourier}$$

$$\rightarrow J = L_{\mu\mu}\left(-\frac{1}{T}\frac{\partial \mu}{\partial x}\right), \quad \mu = c, \quad L_{\mu\mu} = DT \rightarrow \text{Fick}$$

$$j = L_{\phi\phi}\left(-\frac{1}{T}\frac{\partial \phi}{\partial x}\right), \quad L_{\phi\phi} = \frac{T}{R} \rightarrow \text{Ohm}$$

Oppg. 3a)

$J_s(x) \rightarrow \quad \sigma(x) \quad \rightarrow J_s(x+dx)$

$x \qquad\qquad x+dx$

Akkumulert = Inn - Ut + Dannet

$$\frac{dS}{dt} = AJ_s(x) - AJ_s(x+dx) + \sigma(x) \quad , \quad [\sigma] = \frac{J}{Ks}$$

omdef. σ

$$\frac{ds}{dt} = -\frac{A(J_s(x+dx) - J_s(x))}{A\,dx} + \sigma(x) \quad , \quad [\sigma] = \frac{J}{m^3 Ks}$$

$$s = \frac{S}{dV} \quad , \quad dV = A\,dx$$

$$\underline{\frac{ds}{dt} = -\frac{\partial}{\partial x} J_s + \sigma}$$

3b)

$$\frac{dc_i}{dt} = -\frac{\partial}{\partial x} J_i + \nu_i r$$

$$\frac{dz_i}{dt} = -\frac{\partial}{\partial x} j + \sum_i z_i \nu_i r$$

$$\frac{dU}{dt} = -\frac{\partial}{\partial x} J_q - j\frac{\partial \phi}{\partial x}$$

3c)

$$dU = TdS - pdV + \sum_i \mu_i dn_i \quad , \quad \Omega = \omega V$$

$$\Omega = \{U, S, n\}$$

$$d(uV) = Td(sV) - pdV + \sum \mu_i d(c_i V)$$

$$udV + Vdu = T(sdV + Vds - pdV) + \sum \mu_i (c_i dV + Vdc_i)$$

$$(du - Tds - \sum \mu_i dc_i)V = \underbrace{(Ts - p + \sum \mu_i c_i - u)}_{=0} dV$$

$$\underline{du = Tds + \sum \mu_i dc_i}$$

3d) $\frac{ds}{dt} = -\frac{\partial}{\partial x} J_s + \sigma$

$$\frac{du}{dt} = T\frac{ds}{dt} + \sum \mu_i \frac{dc_i}{dt} = -\frac{\partial}{\partial x} J_q - j\frac{\partial \phi}{\partial x}$$

$$T\frac{ds}{dt} = -\frac{\partial}{\partial x} J_q - j\frac{\partial \phi}{\partial x} - \sum \mu_i\left(-\frac{\partial}{\partial x} J_i + \nu_i r\right)$$

$$\frac{ds}{dt} = \frac{1}{T}\left(-\frac{\partial}{\partial x} J_q - \frac{j}{T}\frac{\partial \phi}{\partial x} + \sum \frac{\mu_i}{T}\frac{\partial}{\partial x} J_i + r\Delta_{rxn} G\right)$$

$$= J_q \frac{\partial}{\partial x}\left(\frac{1}{T}\right) - \frac{\partial}{\partial x}\left(\frac{J_q}{T}\right) - \frac{j}{T}\frac{\partial \phi}{\partial x} + \sum \frac{\partial}{\partial x}\left(\frac{\mu_i J_i}{T}\right) - J_i \frac{\partial}{\partial x}\left(\frac{\mu_i}{T}\right) + r\Delta_{rxn} G$$

$$\frac{ds}{dt} = \frac{\partial}{\partial x}\underbrace{\left(-\frac{J_q}{T} + \frac{\mu_i J_i}{T}\right)}_{-J_s} \underbrace{- \frac{j}{T}\frac{\partial \phi}{\partial x} + \sum J_i \frac{\partial}{\partial x}\left(-\frac{\mu_i}{T}\right) + J_q \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + r\Delta_{rxn} G}_{\sigma}$$

$$\sigma = J_q \frac{\partial}{\partial x}\left(\frac{1}{T}\right) - \sum J_i \frac{\partial}{\partial x}\left(\frac{\mu_i}{T}\right) - \frac{j}{T}\frac{\partial \phi}{\partial x} + r\Delta_{rxn} G \quad \square$$

3e) Assume local mechanical equilibrium (and thermal?) (and electrical?) (not chemical at least)

3d) $\sum J_i X_i \geq 0$

because $\sigma \geq 0$ (entropy can never be consumed)

Oppg. 4a) J_q includes heat transported as enthalpy

4b) $$\frac{\partial}{\partial x}\left(\frac{\mu_i}{T}\right) = \mu_i \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \frac{1}{T}\left(\frac{\partial \mu_{i,T}}{\partial x} + \frac{\partial \mu_i}{\partial T}\frac{\partial T}{\partial x}\right)$$

$$= (h_i - Ts_i)\frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \frac{1}{T}\frac{\partial \mu_{i,T}}{\partial x} - \frac{1}{T^2}\left(Ts_i \frac{\partial T}{\partial x}\right) \qquad \frac{\partial}{\partial x}\left(\frac{1}{T}\right) = \frac{-1}{T^2}\frac{\partial T}{\partial x}$$

$$= (h_i - Ts_i)\frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \frac{1}{T}\frac{\partial \mu_{i,T}}{\partial x} + Ts_i \frac{\partial}{\partial x}\left(\frac{1}{T}\right)$$

$$= h_i \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \frac{1}{T}\frac{\partial \mu_{i,T}}{\partial x}$$

$$\sigma = J_q \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \sum J_i\left(-\frac{\partial}{\partial x}\left(\frac{\mu_i}{T}\right)\right) + B$$

$$= \left(J_q' + \sum h_i J_i\right)\frac{\partial}{\partial x}\left(\frac{1}{T}\right) - \sum J_i\left[h_i \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \frac{1}{T}\frac{\partial \mu_{i,T}}{\partial x}\right]$$

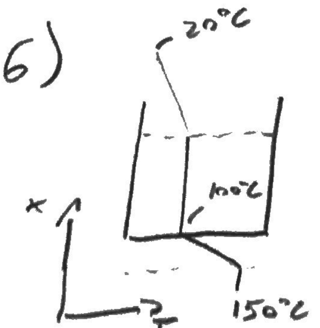
$$\sigma = J_q' \frac{\partial}{\partial x}\left(\frac{1}{T}\right) + \sum J_i\left(-\frac{1}{T}\frac{\partial \mu_{i,T}}{\partial x}\right) \quad \square$$

Oppg. 5) $\sigma = -\frac{j}{T}\frac{\partial \phi}{\partial x}$, $\frac{\partial T}{\partial x} = \frac{\partial \mu}{\partial x} = 0$

$$\rightarrow \frac{\partial \phi}{\partial x} = -\frac{1}{\kappa} j$$

$$\sigma = \frac{j^2}{\kappa T} = \frac{2}{3}\frac{J}{K m^3}$$

Oppg. 6)



Assuming steady state heat transfer through the bottom

$$Q = A k \frac{\Delta T}{\Delta x} = 36 \frac{kJ}{s} = A J_q$$

$$\sigma = J_q \frac{\partial}{\partial x}\left(\frac{1}{T}\right)$$

$$\frac{dS_{irr}}{dt} = A\int_{\text{plate}}^{\text{surrounding}} \sigma \, dx = A\int_p^s J_q \frac{\partial}{\partial x}\left(\frac{1}{T}\right)dx = A J_q \int_{423}^{293} \frac{\partial}{\partial T}\left(\frac{1}{T}\right)dT$$

$$\frac{dS_{irr}}{dt} = A J_q\left(\frac{1}{293K} - \frac{1}{423K}\right) = \underline{\underline{37{,}8 \frac{J}{s}}}$$