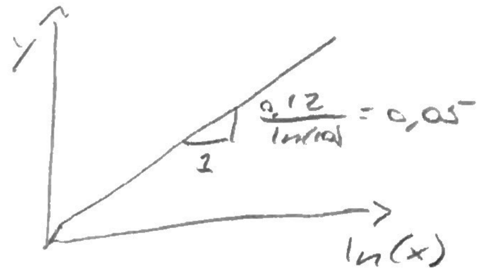
Elektro, øving 2, Vegard G. Jervell

Oppg 1a) $y(x) = a + b\ln(x)$

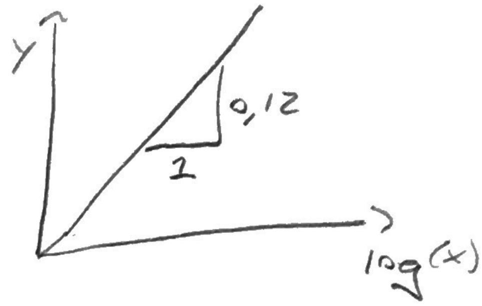
$b\ln(x) = b\ln 10^{\log(x)} = b'\log(x)$, $\underline{b' = b\ln(10)}$

$y(x) = a + b\ln(10)\log(x)$

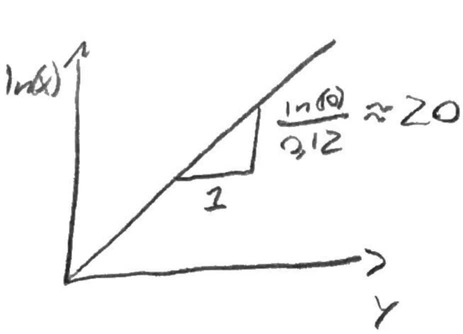
1b)



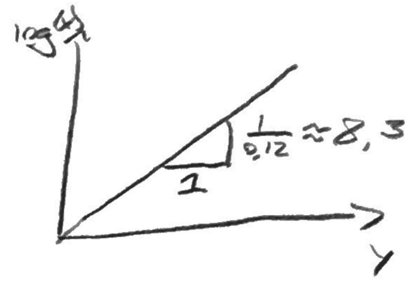
1c)



1d)



1e



1f) $y = e^{(1-a)x} - e^{-ax} \xrightarrow[x \to 0]{\text{Taylor}} (1-a)x - (-a)x$

$\lim_{x \to 0} y(x) = x$

Oppg 2) $U = RI \to \underline{U = 30V}$

Oppg 3) $U = RI \to \underline{I = \frac{V}{R} = 2A}$

Oppg. 4)

$$R_T = \sum R_i = 5\,\Omega$$

$$I = \frac{U}{R_T} = 0{,}8A$$

$$U_1 = -R_1 I = -3\Omega\, 0{,}8A = -2{,}4V$$

$$U_2 = -R_2 I = -1{,}6V$$

Oppg. 5)

$$\frac{d^2\Phi}{dx^2} = -\frac{\rho}{\varepsilon}$$

$$\frac{d\Phi}{dx} = -\frac{\rho}{\varepsilon}x + A$$

$$\Phi(x) = -\frac{\rho}{2\varepsilon}x^2 + Ax + B$$

$$\Phi(x_1) = \Phi_1 \rightarrow -\frac{\rho}{2\varepsilon}x_1^2 + Ax_1 + B = \Phi_1$$

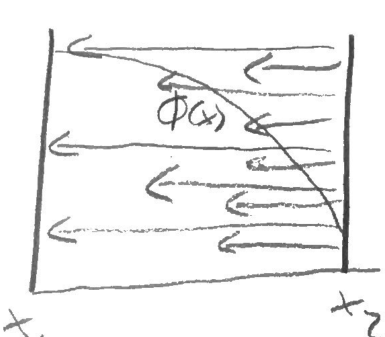
$$\Phi(x_2) = \Phi_2 \rightarrow -\frac{\rho}{2\varepsilon}x_2^2 + Ax_2 + B = \Phi_2$$

$$-\frac{\rho}{2\varepsilon}(x_1^2 - x_2^2) + A(x_1 - x_2) = \Phi_1 - \Phi_2$$

$$A = \frac{\Phi_1 - \Phi_2 + \frac{\rho}{2\varepsilon}(x_1^2 - x_2^2)}{x_1 - x_2}$$

Velger $B = 0$, fordi man kan velge nullpunkt for potensialet fritt

$$\Phi(x) = -\frac{\rho}{2\varepsilon}x^2 + \frac{\Phi_1 - \Phi_2 + \frac{\rho}{2\varepsilon}(x_1^2 - x_2^2)}{x_1 - x_2}x \quad , \vec{E}(x) = \nabla\Phi(x)$$



$|\vec{E}(x)|$ øker med økende x

feltstyrken

Oppg. 9a)

$$C = \frac{Q}{\Delta E} \rightarrow \Delta E = \frac{Q}{C} = 1000V$$

9b) $C = \frac{\varepsilon A}{l}$ Hvis avstanden dobles, halveres kapasitansen.

$$C_1 = \frac{\varepsilon A}{l_1}, \quad C_2 = \frac{\varepsilon A}{2l_1} = \frac{1}{2}C_1$$

$$\Delta E = \frac{Q}{\frac{1}{2}C_1} = 2\Delta E_1 = 2kV$$

Oppg. 10a) $i = kX, \quad X = \frac{\Delta E}{l}, \quad I = \frac{\Delta E}{R}, \quad I = Ai$

$$\frac{I}{A} = k\frac{\Delta E}{l}$$

$$\frac{\Delta E}{RA} = k\frac{\Delta E}{l}$$

$$R = \frac{l}{Ak} = 2{,}31 \cdot 10^3 S^{-1} = 2{,}31 k\Omega$$

10b) $R_1 = \frac{l_1}{Ak} = \frac{0{,}05cm}{0{,}1cm^2 \; 0{,}013Scm^{-1}} = \underline{38{,}4\,\Omega}$

$R_2 = \frac{l_2}{Ak}, \quad l_2 = 0{,}1cm \rightarrow R_2 = \underline{76{,}9\,\Omega}$

$l_3 = 1cm \rightarrow R_3 = 769\,\Omega$

Oppg. 11a) $\mathcal{L}(3t+4) = \frac{3}{s^2} + \frac{4}{s}$

$$\mathcal{L}(t^2+at+b) = \frac{2}{s^3} + \frac{a}{s^2} + \frac{b}{s}$$

$$\mathcal{L}\left(\frac{5}{\sqrt{\pi t}}\right) = \frac{5}{\sqrt{s}}$$

11b) $\mathcal{L}^{-1}\left(\frac{5}{s+3}\right) = 5e^{-3t}$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2+25}\right) = \sin(5t)$$

$$\mathcal{L}^{-1}\left(\frac{2}{s}\exp\left[-\sqrt{\frac{s}{\pi}}\right]\right) = 2\,\mathrm{erfc}\left(\frac{1}{2\sqrt{\pi t}}\right)$$

Prob. 12)

$$\frac{dy}{dt} + \frac{1}{\tau} y = K$$

$$s Y(s) - y(0) + \frac{1}{\tau} Y(s) = \frac{K}{s}$$

$$Y(s) = \frac{\frac{K}{s} + y(0)}{s + \frac{1}{\tau}}$$

$$Y(s) = \frac{K}{s^2 + \frac{s}{\tau}} + \frac{y_0}{s + \frac{1}{\tau}}$$

$$\frac{K}{s(s+\frac{1}{\tau})} = \frac{A}{s} + \frac{B}{s+\frac{1}{\tau}}$$

$$K = A(s + \frac{1}{\tau}) + Bs$$

$$A + B = 0$$

$$A(\frac{1}{\tau}) = K$$

$$A = K\tau \rightarrow B = -K\tau$$

$$Y(s) = \frac{K\tau}{s} - \frac{K\tau}{s+\frac{1}{\tau}} + \frac{y_0}{s+\frac{1}{\tau}}$$

$$y(t) = K\tau - K\tau e^{-\frac{t}{\tau}} + y_0 e^{-\frac{t}{\tau}}$$