Statistikk, Innlevering 4, Vegard G. Jervell

Oppg. 1a)

$$X \sim N(\mu, \sigma^2)$$

$$P(x > \mu + \sigma) = P\left(\frac{X-\mu}{\sigma} > 1\right)$$

$$Z := \frac{X-\mu}{\sigma} \sim N(0,1)$$

$$P\left(\frac{X-\mu}{\sigma} > 1\right) = 1 - P(Z \leq 1) = 1 - 0{,}8413$$

$$\underline{\underline{P(X > 10{,}2g) = 0{,}1587}}$$

$$Y := \frac{X_1 + X_2}{2} \rightarrow \sigma_Y = \frac{\sigma_X}{\sqrt{2}}, \quad \mu_Y = \mu_X$$

$$P(|Y - \mu_X| > \sigma_X) = P(Y - \mu_X < -\sigma_X) + P(Y - \mu_X > \sigma_X)$$

$$= P(Y < \mu_X - \sigma_X) + P(Y > \mu_X + \sigma_X)$$

$$Z := \frac{Y - \mu_Y}{\sigma_Y} = \frac{Y - \mu_X}{\frac{\sigma_X}{\sqrt{2}}} \sim N(0,1)$$

$$P(|Y - \mu_X| > \sigma_X) = P\left(Z < \frac{-\sigma_X}{\frac{\sigma_X}{\sqrt{2}}}\right) + P(Z > \sqrt{2})$$

$$= 2 \cdot P(Z < -\sqrt{2})$$

$$\underline{\underline{= 0{,}1486}}$$

Oppg 1b) $\hat{\mu}_A = X_1 \to E(\hat{\mu}_A) = E(X_1) = 10g$

$E(\hat{\mu}_B) = 10g$

$Var(\hat{\mu}_A) = Var(X_1) = 0{,}2^2 g^2$

$Var(\hat{\mu}_B) = 0{,}2^2 g^2$

$Y_1 := X_1 + X_2 \qquad Y_2 = X_1 - X_2$

$\tilde{\mu}_A := \frac{Y_1 + Y_2}{2} \qquad \tilde{\mu}_B := \frac{Y_1 - Y_2}{2}$

$E(\tilde{\mu}_A) = \frac{1}{2} E(Y_1 + Y_2) = \frac{1}{2} E(2X_1) = 10g$

$E(\tilde{\mu}_B) = \frac{1}{2} E(Y_1 - Y_2) = E(X_2) = 10g$

$$Var(\tilde{\mu}_A) = Var\left(\frac{Y_1 + Y_2}{2}\right) = \frac{1}{4} Var(Y_1 + Y_2)$$
$$= \frac{1}{4}\left(Var(Y_1) + Var(Y_2) + Cov(Y_1, Y_2)\right)$$

$Var(Y_1) = Var(X_1) + Var(X_2) = 2\sigma_x^2$

$Var(Y_2) = Var(X_1) + (-1)^2 Var(X_2) = 2\sigma_x^2$

$$cov(Y_1, Y_2) = E(Y_1 Y_2) - E(Y_1)\underbrace{E(Y_2)}_{=0}$$
$$= E((X_1 + X_2)(X_1 - X_2))$$
$$= E(X_1^2) - E(X_2^2)$$
$$= 0$$

$\to Var(\tilde{\mu}_A) = \sigma_x^2$ dette er feil, men vet ikke hvorfor :($Var[\sum a_i X_i] = \sum a_i^2 Var[X_i]$

Gitt at fasit er riktig og
$Var(\tilde{\mu}_A) = \frac{1}{2}\sigma_x^2$ ville jeg brukt
$\tilde{\mu}_A$ fordi mindre varians og den
er forventningsrett.

Oppg. 2a)

$$P(Z>10) = 1 - P(Z \leq 10)$$
$$= 1 - F(10, \lambda)$$
$$\underline{P(Z>10) = 0,61}$$

$$P(Z>20 \mid Z>10) = \frac{P(Z>20)}{P(Z>10)} = \frac{e^{-1}}{e^{-0,5}} = 0,61$$

2b)

$Z < 1 \rightarrow M=0 \quad P(M=0) = F(1, \lambda)$

$1 < Z < 2 \rightarrow M=1 \quad P(M=1) = P(1<Z<2) = F(2,\lambda) - F(1,\lambda)$

$m < Z < m+1 \rightarrow M=m \quad P(M=m) = P(m<Z<m+1)$

$$= F(m+1, \lambda) - F(m)$$
$$= 1 - e^{-\lambda(m+1)} - (1 - e^{-\lambda m})$$
$$= -e^{-\lambda} e^{-\lambda m} + e^{-\lambda m}$$
$$P(M=m) = e^{-\lambda m}(1 - e^{-\lambda}) \quad \square$$

2c) $L(m_1, m_2, \ldots, m_n; \lambda) = \prod_{i=1}^{n} (1-e^{-\lambda}) e^{-\lambda m_i} = (1-e^{-\lambda})^n \exp\left(-\lambda \sum_{i=1}^{n} m_i\right)$

$$\begin{aligned} l(\vec{m};\lambda) &= \ln(L(\vec{m};\lambda)) \\ &= n \ln(1-e^{-\lambda}) - \lambda \sum_{i=1}^{n} m_i \end{aligned}$$

$$\frac{dl}{d\lambda} = 0 \rightarrow \frac{ne^{-\lambda}}{1-e^{-\lambda}} - \sum_{i=1}^{n} m_i = 0$$

$$ne^{-\lambda} = (1-e^{-\lambda}) \sum m_i$$

$$e^{-\lambda}(n + \sum m_i) = \sum m_i$$

$$\underline{\lambda = \ln\left(\frac{n+\sum m_i}{\sum m_i}\right)}$$

$$\frac{d^2 l}{d\lambda^2} = n\left(\frac{-e^{-\lambda}}{1-e^{-\lambda}} + e^{-\lambda}\left(e^{-\lambda}(1-e^{-\lambda})^2\right)\right)$$

$$= \frac{-ne^{-\lambda}}{(1-e^{-\lambda})^2} < 0 \rightarrow \lambda = \ln\left(\frac{n}{\sum m_i} + 1\right)$$

er maks

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oppg 3) kumulativ sannsynlighet for at bare komponent i dør er

$$F_{X_i}(x) \prod_{j \neq i} (1 - F_{X_j}(x))$$

komp. i dør

ingen andre dør

Sannsynlighet for at en vilkårlig komp. dør

$$P(\text{en død}) = \sum_i F_{X_i}(x) \prod_{j \neq i} (1 - F_{X_j}(x))$$

Binomisk fordelt

$$\begin{aligned} P(\text{èn død}) &= n F_X(x) (1 - F_X(x))^{n-1} \\ &= n(1 - e^{-(\lambda x)^\alpha}) e^{(1-n)(\lambda x)^\alpha} \\ &= n(e^{(1-n)(\lambda x)^\alpha} - e^{-n(\lambda x)^\alpha}) \end{aligned}$$

Oppg. 4)

$f(x) = \lambda e^{-\lambda x}, x>0 \quad Y = X_1 + X_2$

Skal vise: $f(y) = \lambda^2 y e^{-\lambda y}, \quad y > 0$

Finner kumulativ sannsynlighet for Y

$$P(Y < y) = P(X_1 + X_2 < y) = P(X_2 < y - X_1)$$

$$= \int_0^y \int_0^{y-x_1} f(x_1) f(x_2)\, dx_2\, dx_1$$

$$= \int_0^y \lambda^2 e^{-\lambda x_1} \frac{-1}{\lambda} \left[e^{-\lambda x_2}\right]_0^{y-x_1} dx_1$$

$$= -\lambda \int_0^y e^{-\lambda x_1} \left(e^{-\lambda(y-x_1)} - 1\right) dx_1$$

$$= -\lambda \int_0^y e^{-\lambda y} - e^{-\lambda x_1} dx_1$$

$$P(Y<y) = -\lambda y e^{-\lambda y} - e^{-\lambda y} + 1$$

$$f(y) = \frac{dP(Y<y)}{dy} = \lambda^2 y e^{-\lambda y} - \lambda e^{-\lambda y} + \lambda e^{-\lambda y}$$

$$f(y) = \lambda^2 y e^{-\lambda y} \quad \square$$