Termo, Øving 5, Vegard G. Jervell

T.1) Det totale differensialet til en funksjon $F(x,y,z)$ er gitt ved:

$$dF(x,y,z) = \left(\frac{\partial F}{\partial x}\right)dx + \left(\frac{\partial F}{\partial y}\right)dy + \left(\frac{\partial F}{\partial z}\right)dz$$

T.2) Fordi de gjør at vi kan regne på ting som henger sammen

T.3) $dH = \left(\frac{\partial H}{\partial U}\right)dU + \left(\frac{\partial H}{\partial p}\right)dp + \left(\frac{\partial H}{\partial V}\right)dV$

$$dS = \left(\frac{\partial S}{\partial T}\right)dT + \left(\frac{\partial S}{\partial V}\right)dV$$

$$dU = \left(\frac{\partial U}{\partial T}\right)dT$$

T.4) Entropi kan måles ved å måle varmekapasitet som funksjon av T, samt smelte- og fordampningsentalpier. Deretter definere $S(0K) = 0$ og beregne

$$\Delta S = \int_0^{T'} \frac{C_p}{T}\,dT$$

1) $$\Delta S = \int_{T_0}^{T_K} \frac{C_p(l)}{T}\,dT + \frac{\Delta_{vap}H}{T_{vap}} + \int_{T_{vap}}^{T_1} \frac{C_p(g)}{T}\,dT$$

$$\Delta S = C_p(l)\ln\left(\frac{T_{vap}}{T_0}\right) + \frac{\Delta_{vap}H}{T_{vap}} + a\ln\left(\frac{T_1}{T_{vap}}\right) + b(T_1 - T_{vap}) + \frac{1}{2}c\left(T_1^2 - T_{vap}^2\right)$$

$$\underline{\underline{\Delta S = 124\,\frac{J}{K\cdot mol}}}$$

2.1) $\Delta S = \frac{-\Delta_{fus} H(-10°C)}{T_{fus}} = \frac{-5418 \frac{J}{mol}}{263 K} = -20,6 \frac{J}{mol \cdot K}$, $\Delta_{fus} H(T) = \Delta_{fus} H(T_{ref}) + \Delta_{fus} C_p \Delta T$

2.2)

$$\underset{\substack{H_2O(l)\\-10°C}}{A} \xrightarrow{1} \underset{\substack{H_2O(l)\\0°C}}{B} \xrightarrow{2} \underset{\substack{H_2O(s)\\0°C}}{C} \xrightarrow{3} \underset{\substack{H_2O(s)\\-10°C}}{D}$$

$$q_1 = -\int_{T_A}^{T_B} C_{p,l}\, dT \quad , \quad q_2 = \Delta_{fus} H \quad , \quad q_3 = -\int_{T_C}^{T_D} C_{p,s}\, dT \, .$$

$$\Delta S_{om} = \frac{\sum q_i}{T_{om}} = 21,4 \frac{J}{K}$$

3.1) $dF = X(x,y,z)dx + Y(x,y,z)dy + Z(x,y,z)dz$

er et totalt differensial av $F(x,y,z)$

når

$$X = \frac{\partial F}{\partial x} \wedge Y = \frac{\partial F}{\partial y} \wedge Z = \frac{\partial F}{\partial z}$$

3.2) $dp = \frac{nR}{V} dT + \frac{nRT}{V^2} dV$

$$\frac{\partial^2 p}{\partial T \partial V} = -\frac{nR}{V^2} \neq \frac{nR}{V^2}$$, ikke denne :(

ii) $dp = \frac{nR}{V} dT - \frac{nRT}{V^2} dV$

$$\frac{\partial p}{\partial T} = \frac{nR}{V} \rightarrow p = \frac{nRT}{V} + C_1(V)$$

$$\frac{\partial p}{\partial V} = -\frac{nRT}{V^2} \rightarrow p = \frac{nRT}{V} + C_2(T)$$

$\rightarrow C_1 = C_2 = 0$

$$p = \frac{nRT}{V}$$

3.3) bare (i) har en grense som eksisterer og gir $U = U_{ideell}$ når $(a,b) \to (0,0)$

(i) er riktig

3.4) $\frac{\partial U}{\partial V} = \frac{an^2}{V^2}$, $\frac{\partial U}{\partial T} = \frac{3}{2} nR$

$$dU = \frac{an^2}{V^2} dV + \frac{3}{2} nR\, dT$$

$$\frac{\partial U}{\partial p} = \frac{\partial}{\partial p}\left[\frac{3}{2}\left(p + \frac{an^2}{V^2}\right)(V - nb) - \frac{an^2}{V}\right]$$

$$\frac{\partial U}{\partial p} = \frac{3}{2}(V - nb)$$

$$\frac{\partial U}{\partial V} = \frac{3}{2}\left(-\frac{an^2}{V^2} + \frac{2abn^3}{V^3} + \frac{an^2}{V^2}\right)$$

$$dU = \frac{3}{2}(V - nb)\,dp + \frac{3abn^3}{V^3} dV$$