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$$1) F(f') = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f'(x) e^{-iwx} dx = \frac{1}{\sqrt{2\pi}} \left[f(x) e^{-iwx} \right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} -iwx f(x) e^{-iwx} dx$$

$$F(f') = \frac{iw}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx = iw F(f) \quad \square$$

$$3) F(e^{-|x|}) = \frac{1}{\sqrt{2\pi}} \left(\int_{-\infty}^0 e^x e^{-iwx} dx + \int_0^{\infty} e^{-x} e^{-iwx} dx \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{1}{1-iw} e^{x(1-iw)} \Big|_{-\infty}^0 + \frac{1}{-1-iw} e^{x(-1-iw)} \Big|_0^{\infty} \right)$$

$$= \frac{1}{\sqrt{2\pi}} \left(\frac{1}{1-iw} + \frac{1}{1+iw} \right)$$

$$F(e^{-|x|}) = \frac{1}{\sqrt{2\pi}} \left(\frac{2}{1+w^2} \right)$$

$$4) f(x) = x^2 e^{-x^2}, \quad g(x) = e^{-x^2}$$

$$\frac{dg}{dx} = -2x e^{-x^2}$$

$$\frac{d^2 g}{dx^2} = -2e^{-x^2} + 4x^2 e^{-x^2}$$

$$F(g'') = -2F(g) + 4F(f)$$

$$-w^2 F(g) + 2F(g) = 4F(f)$$

$$\frac{1}{\sqrt{2}} \exp\left(-\frac{w^2}{4}\right) (2-w^2) = 4F(f)$$

$$F(f) = \frac{2-w^2}{4\sqrt{2}} \exp\left(-\frac{w^2}{4}\right)$$

$$5) f(x) = e^{-x^2}, \quad g(x) = xe^{-x^2}$$

$$f * g = F^{-1}(F(f * g)) = F^{-1}(\sqrt{2\pi} F(f) F(g))$$

$$F(f) = \frac{1}{\sqrt{2}} \exp\left(-\frac{w^2}{4}\right)$$

$$F'(x) = -2x e^{-x^2}$$

$$F(f') = -2 F(g)$$

$$i w F(f) = -2 F(g)$$

$$F(g) = -\frac{i w}{2} F(f) = \frac{-i w}{2 \sqrt{2}} \exp\left(-\frac{w^2}{4}\right)$$

$$\sqrt{2\pi} F(f) F(g) = \sqrt{2\pi} \left(\frac{-i w}{4} \exp\left(-\frac{w^2}{2}\right) \right)$$

$$f * g = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{-i w \sqrt{2\pi}}{4} \exp\left(-\frac{w^2}{2}\right) \exp(i w x) dx$$

$$f * g = -\frac{i}{4} \int_{-\infty}^{\infty} w e^{-\frac{w^2}{2}} e^{i w x} dx \quad \square$$
