1) Dersom $F(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}$

$$F(x) \cdot F(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx} \sum_{m=-\infty}^{\infty} C_m e^{imx}$$

$$(F(x))^2 = \sum_{n=-\infty}^{\infty} C_n e^{inx} \sum_{m=-\infty}^{\infty} C_{-m} e^{-imx}$$

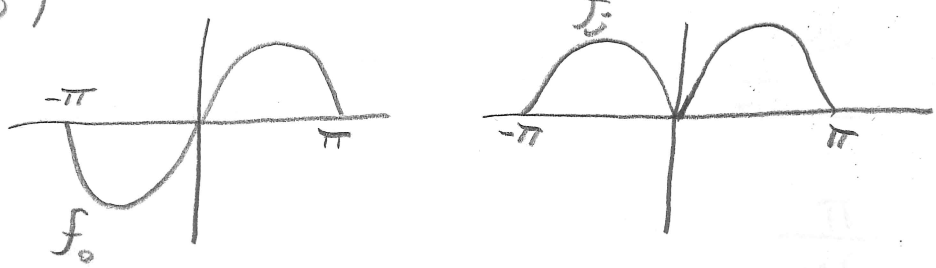
$$(F(x))^2 = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} C_n C_{-m} e^{i(n-m)x}$$

$$\int_{-\pi}^{\pi} (F(x))^2 \, dx = 2\pi \sum_{n=-\infty}^{\infty} |C_n|^2 \quad \square$$

Fordi $C_n = \overline{C_{-n}}$

og $\int_{-\pi}^{\pi} e^{i(n-m)x} \, dx = \begin{cases} 0, & n \neq m \\ 2\pi, & n = m \end{cases}$

3)



Fourier-rekken til $\underline{f_o = \sin(x)}$

Fourier-rekken til f_j:

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F_j \cos(nx) \, dx = \frac{2}{\pi} \int_{0}^{\pi} F(x) \cos(nx) \, dx$$

$$a_n = \frac{2}{\pi} \left(\left[\cos(x) \cos(nx) \right]_0^{\pi} - \int_0^{\pi} +n \cos(x) \sin(nx) \, dx \right.$$

$$a_n = \begin{cases} 0, & n \text{ er odde} \\ \frac{2}{1-n^2}, & n \text{ er par} \end{cases}$$

$$f_j \sim \sum_{n=1}^{\infty} \frac{2}{1-4n^2} \cos(nx)$$

4) $f(x) = x^2 \quad , \qquad -1 \leq x \leq 1$

$$a_0 = \frac{1}{2}\int_{-1}^{1} x^2 dx = \frac{1}{3}$$

$$a_n = \int_{-1}^{1} x^2 \cos(n\pi x) dx = \frac{2}{(n\pi)^2}\left((-1)^n + (-1)^n\right)$$

$$b_n = \int_{-1}^{1} x^2 \sin(n\pi x) dx = \frac{2-(n\pi)^2}{(n\pi)^3}(-1)^n - \frac{2-(n\pi)^2}{(n\pi)^3}(-1)^n = 0$$

$$x^2 \sim \frac{1}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{(n\pi)^2}\cos(nx)$$

Parceval:

$$\frac{1}{2}\int_{-1}^{1}(x^2)^2 dx = \left(\frac{1}{3}\right)^2 + \frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{(n\pi)^2}\right)^2$$

$$\frac{1}{2}\int_{-1}^{1} x^4 dx = \frac{1}{9} + \frac{8}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\left(\frac{1}{5} - \frac{1}{9}\right)\frac{\pi^4}{8} = \sum_{n=1}^{\infty}\frac{1}{n^4}$$

$$\underline{\underline{\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}}}$$