Matte 4, Øving 3. Vegard G. Jervell

Oppg. 1)

$$F(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}$$

$$e^{-imx} F(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx} e^{-imx}$$

$$\int_{-\pi}^{\pi} e^{-imx} F(x)\,dx = \sum_{n=-\infty}^{\infty} \int_{-\pi}^{\pi} C_n e^{inx} e^{-imx}\,dx$$

$$\int_{-\pi}^{\pi} e^{-imx} F(x)\,dx = C_m \cdot 2\pi$$, Fordi e^{inx} og e^{-imx} er ortogonale

$$C_m = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-imx} F(x)\,dx \quad \square$$

Oppg. 3)

$$F(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ x(\pi - x), & 0 \leq x \leq \pi \end{cases}$$

$$F(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(x)\,dx$$

$$\underline{\underline{a_0 = \frac{1}{2\pi}\left(\frac{1}{2}\pi^3 - \frac{1}{3}\pi^3\right) = \frac{\pi^2}{12}}}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x)\cos(nx)\,dx$$

$$\int_0^{\pi} x\cos(nx)\,dx = \underline{\frac{(-1)^n - 1}{n^2}} \qquad \int_0^{\pi} x^2\cos(nx)\,dx = \underline{\frac{2\pi(-1)^n}{n^2}}$$

$$\int_0^{\pi} x\sin(nx)\,dx = \underline{\frac{-\pi(-1)^n}{n}} \qquad \int_0^{\pi} x^2\sin(nx)\,dx = \underline{\frac{2 - n^2\pi^2}{n^3}(-1)^n - \frac{2}{n^3}}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} x(\pi - x)\cos nx\,dx = \frac{(-1)^n - 1}{n^2} - \frac{2(-1)^n}{n^2} = \frac{(-1)^{n+1} - 1}{n^2}$$

$$\underline{a_n = \begin{cases} 0, & n \text{ er odde} \\ \frac{-2}{n^2}, & n \text{ er par} \end{cases}}$$

$$b_n = \frac{1}{\pi}\int_0^{\pi} x(\pi - x)\sin(nx)\,dx = \frac{\pi(-1)^{n+1}}{n} - \frac{1}{\pi}\left(\frac{2 - n^2\pi^2}{n^3}(-1)^n - \frac{2}{n^3}\right)$$

$$= \frac{2(-1)^{n+1} + 2}{\pi n^3}$$

$$b_n = \begin{cases} 0, & n \text{ er par} \\ \frac{4}{\pi n^3}, & n \text{ er odde} \end{cases}$$

$$\underline{\underline{F(x) \sim \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{-1}{2n^2}\cos(2nx) + \sum_{n=1}^{\infty} \frac{4}{\pi(2n-1)^3}\sin((2n-1)x)}}$$