# Øving 1

Oppg. 1)

$L(F) = \int_0^\infty F(t)e^{-st}\,dt$ konvergerer når

$s > a$ og $|F(t)| \leq Me^{at}$ for $t \geq 0$

hvor $a$ og $M > 0$ er reele.

$$\left|\int_0^\infty F(t)e^{-st}\,dt\right| \leq \left|\int_0^\infty Me^{at}e^{-st}\,dt\right| = M\int_0^\infty e^{(a-s)t}\,dt$$

$$\left|\int_0^\infty F(t)e^{-st}\,dt\right| \leq \frac{M}{a-s}\left[e^{(a-s)t}\right]_0^\infty = \frac{M}{s-a}$$

Oppg. 3a)

$$\int_0^\infty \sinh(t)\cos(t)e^{-st}\,dt = \int_0^\infty \left(\frac{e^t-e^{-t}}{2}\right)\left(\frac{e^{it}+e^{-it}}{2}\right)e^{-st}\,dt$$

$$= \frac{1}{4}\int_0^\infty \left(e^{(1+i)t} + e^{(1-i)t} - e^{(-1+i)t} - e^{(-1-i)t}\right)e^{-st}\,dt$$

$$= \frac{1}{4}\left(\frac{-1}{1+i-s} + \frac{-1}{1-i-s} + \frac{1}{-1+i-s} + \frac{1}{-1-i-s}\right) = \underline{\underline{\frac{s^2-2}{s^4+4}}}$$

3b)

$$\int_0^\infty \cos^2(2t)e^{-st}\,dt = \frac{1}{4}\int_0^\infty \left(e^{2it}+e^{-2it}\right)^2 e^{-st}\,dt$$

$$= \frac{1}{4}\int_0^\infty e^{(4i-s)t} + e^{(-4i-s)t}\,dt$$

$$= \frac{1}{4}\left(\frac{1}{s-4i} + \frac{1}{s+4i}\right)$$

$$= \frac{1}{4}\left(\frac{2s}{s^2+4}\right) = \underline{\underline{\frac{s}{2(s^2+4)}}}$$

Oppg 2)

```
import matplotlib.pyplot as plt, numpy as np
x_list = np.linspace(-np.pi, np.pi, 1000)
y_list = [np.cos(3*x) * np.cos(2*x) for x in x_list]
plt.plot(x_list, y_list)
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4a) $\frac{4}{s^2-2s-3} = \frac{4}{(s-3)(s+1)} = \frac{A}{s-3} + \frac{B}{s+1}$

$4 = A(s+1) + B(s-3)$

$A+B=0 \to A=-B$

$A-3B=4$

$-4B=4$

$\underline{B=-1 \to A=1}$

$\frac{4}{s^2-2s-3} = \frac{1}{s-3} - \frac{1}{s+1}$

$\underline{\underline{L^{-1}\left(\frac{1}{s-3} - \frac{1}{s+1}\right) = e^{3t} - e^{-t}}}$

4b) $\frac{1}{s^4-s^2} = \frac{1}{(s^2+s)(s^2-s)} = \frac{1}{s(s+1)(s-1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s-1}$

$1 = A(s^2-1) + B(s^2-s) + C(s^2+s)$

I $A+B+C=0$

II $C-B=0 \to B=C$

III $-A=1 \to \underline{A=-1}$

I) $-1+2B=0$

$\underline{B=\frac{1}{2}=C}$

$\frac{1}{s^4-s^2} = \frac{-1}{s} + \frac{1}{2(s+1)} + \frac{1}{2(s-1)}$

$\underline{\underline{L^{-1}\left(\frac{1}{s^4-s^2}\right) = -1 + \frac{1}{2}e^{-t} + \frac{1}{2}e^{t}}}$

5a) $y'' - 3y' + 2y = 0$

$sL(y') - y'(0) - 3(sL(y) - y(0)) + 2L(y) = 0$

$s^2L(y) - sy(0) - y'(0) - 3sL(y) - 3y(0) + 2L(y) = 0$

$L(y)(s^2 - 3s + 2) - sy(0) = y'(0) + 3y(0)$

$$L(y) = \frac{y'(0) + 3y(0) + sy(0)}{s^2 - 3s - 2}$$

$$L(y) = \frac{3 + s}{s^2 - 3s + 2} = \frac{3}{(s-1)(s-2)} + \frac{s}{(s-1)(s-2)}$$

I $3 = A(s-1) + B(s-2)$

$\underline{B = -1,\ A = 1}$

II $s = A(s-1) + B(s-2)$

$A + B = 1 \longrightarrow -B = 1 \rightarrow \underline{B = -1}$

$-A - 2B = 0$ $\quad\quad \underline{A = 2}$

$A = -2B$

$$L(y) = \frac{1}{s-2} - \frac{1}{s-1} + \frac{2}{s-2} - \frac{1}{s-1}$$

$$L(y) = \frac{3}{s-2} - \frac{2}{s-1}$$

$\underline{\underline{y = 3e^{2t} - 2e^{t}}}$

5b) $y'' - 3y' + 2y = e^t$

$s^2 L(y) - sy'(0) - y(0) - 3sL(y) + 3y(0) + 2L(y) = \frac{1}{s-1}$

$L(y)(s^2 - 3s + 2) = \frac{1}{s-1}$

$L(y) = \frac{1}{(s-1)(s-1)(s-2)}$

$\frac{1}{(s-1)^2(s-2)} = \frac{As+B}{(s-1)^2} + \frac{C}{s-2}$

$1 = (As+B)(s-2) + C(s-1)^2$

$A + C = 0 \rightarrow A = -C$

$-2A + B - 2C = 0 \rightarrow B = 0$

$-2B + C = 1 \rightarrow C = 1$

$L(y) = \frac{-s}{(s-1)^2} + \frac{1}{s-2}$

$L(y) = \frac{1}{s-2} - \frac{1}{s-1} - \frac{1}{(s-1)^2}$

$y = e^{2t} - e^t - L^{-1}\left(\frac{1}{(s-1)^2}\right)$

$L^{-1}\left(\frac{1}{(s-1)^2}\right) = \int_0^t e^{\tau} \cdot e^{(t-\tau)} \, d\tau$

$= \int_0^t e^t \, d\tau = \underline{te^t}$

$\underline{\underline{y = e^{2t} - e^t - te^t}}$