(1) $\underline{\underline{A}} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$$A\vec{x} = \lambda \vec{x}$$

$$\text{I} \quad a x_1 + b x_2 = \lambda x_1$$

$$\text{II} \quad c x_1 + d x_2 = \lambda x_2$$

$$\text{I} \quad x_1 (a - \lambda) = -b x_2$$

$$\text{I} \quad x_2 = \frac{x_1 (a-\lambda)}{-b}$$

$$\text{II} \quad c x_1 + x_2 (d - \lambda) = 0$$

$$c x_1 + \frac{x_1 (a-\lambda)}{-b} (d-\lambda) = 0$$

assume $x_1 \neq 0$

$$c + \frac{1}{-b} (a-\lambda)(d-\lambda) = 0$$

$$-cb + ad - \lambda (a+d) + \lambda^2 = 0$$

$$\lambda^2 - \lambda(a+d) = cb - ad$$

$$\left(\lambda - \frac{a+d}{2}\right)^2 = cb - ad + \frac{(a+d)^2}{4}$$

$$\lambda - \frac{a+d}{2} = \left(\frac{1}{4}(a+d)^2 + -\det(A)\right)^{\frac{1}{2}}$$

$$\underline{\underline{\lambda = \frac{a+d}{2} \pm \sqrt{\left(\frac{a+d}{2}\right)^2 - \det(A)}}}$$

$\det(\underline{\underline{B}}) = k \cdot \det(\underline{\underline{A}})$ if $\underline{\underline{B}}$ is generated by multiplying one row of $\underline{\underline{A}}$ by some scalar k.

Because $r\underline{\underline{A}}$ is equivalent to multiplying each of the n rows in $\underline{\underline{A}}$ by r

$\det(r\underline{\underline{A}}) = r^n \cdot \det(\underline{\underline{A}})$

(3) $A^k = A \cdot A^{k-1}$

Because $\det(AB) = \det(A)\det(B)$

$$\det(A^k) = \det(A) \cdot \det(A^{k-1})$$
$$= \det(A) \cdot \det(A) \cdot \det(A^{k-2})$$
$$\vdots$$
$$\underline{\underline{\det(A^k) = (\det(A))^k}}$$

(4) $\det(A^k) = 0$

$(\det(A))^k = 0$

$\det(A) = 0 \longrightarrow A$ is not invertible