(1) if $A \cdot A = A$

and $A^{-1} A = I$

then $(A^{-1} A) \cdot A = A^{-1} (A \cdot A) = IA$

this gives $A^{-1} A = IA$

$$I = IA$$

$$\downarrow$$

$$\underline{\underline{A = I}}$$

(2) $A = \begin{vmatrix} 0 & b \\ a & 0 \end{vmatrix}$

$$A^2 = \begin{vmatrix} ab & 0 \\ 0 & ba \end{vmatrix}$$

$$A^2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \rightarrow b = a^{-1}$$

For all $A = \begin{vmatrix} 0 & a^{-1} \\ a & 0 \end{vmatrix}, \ a \in \mathbb{C}$

$$\underline{\underline{A^2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}}}$$

(3) If $C = AB$ is such that

$C^{-1}C = I$ and $CC^{-1} = I$, then $AB(AB)^{-1} = I$

given that $(AB)^{-1} = B^{-1}A^{-1}$

$$ABB^{-1}A^{-1} = I \qquad\qquad (AB)^{-1} AB = I$$

$$AIA^{-1} = I \qquad\qquad B^{-1}A^{-1}AB = I$$

$$\underline{AA^{-1} = I} \qquad\qquad \underline{A^{-1}A = I}$$

thus, A is invertible if

AB is invertible