C1) $e^{i\theta} \cdot e^{-i\theta} = (\cos\theta + i\sin\theta)(\cos\theta - i\sin\theta)$

$1 = \cos^2\theta + \sin^2\theta$

C2) 1) $e^{z+\pi i} = -e^{z}$

$e^{\pi i} = -1$, <u>er riktig (eulers identitet)</u>

2) $\overline{e^{z}} = e^{\overline{z}}$

$\overline{e^{a+ib}} = e^{\overline{a+ib}}$

$e^{a}(\cos b - i\sin b) = e^{a-ib}$

<u>er riktig (Sin(x) er en odde funksjon)</u>

3) $e^{z} = e^{a}(\cos b + i\sin b)$

$e^{a} \neq 0 \ \forall \ a \in \mathbb{R}$

$\cos b = 0 \rightarrow b = \frac{\pi}{2} + n\pi$

$\sin\left(\frac{\pi}{2}\right) \neq 0 \quad \sin\left(\frac{3\pi}{2}\right) \neq 0$

$\underline{\underline{e^{z} \neq 0 \ \forall \ z \in \mathbb{C}}}$

4) $e^{\frac{\pi}{2}i} = i$ $\quad e^{\frac{\pi}{2}i} = e^{\frac{5\pi}{2}i} \rightarrow$ <u>e^{z} er ikke en-til-en</u>

$e^{\frac{5\pi}{2}i} = i$

5) $e^{-z} = e^{-a-ib} = e^{-a}(\cos(b) - i\sin(b))$

$\frac{1}{e^{z}} = \frac{1}{e^{a}(\cos(b) + i\sin(b))} = e^{-a}\left(\frac{\cos(b) - i\sin(b)}{\cos^2 b + \sin^2(b)}\right)$

$\underline{\underline{e^{-z} = \frac{1}{e^{z}}}}$

$$z^5 = 1$$

$$|z|^5 = 1 \quad \vee \quad 5 \cdot \arg(z) = 0 + n \cdot 2\pi$$

$$|z| = 1 \qquad \arg(z) = \frac{2n\pi}{5}$$

$$\underline{z_1 = e^{0}} \quad \underline{z_2 = e^{i\frac{2\pi}{5}}} \quad \underline{z_3 = e^{i\frac{4\pi}{5}}} \quad \underline{z_4 = e^{i\frac{6\pi}{5}}} \quad \underline{z_5 = e^{i\frac{8\pi}{5}}}$$

$$\cancel{\sum_{n=0}^{4} e^{i\frac{2n\pi}{5}}} \qquad z_4 = \overline{z_3} \qquad z_5 = \overline{z_2}$$

$$z + \overline{z} = 2 \cdot Re(z)$$

$$z_1 + z_2 + z_3 + z_4 + z_5 = z_1 + 2 \cdot Re(z_2) + 2 \cdot Re(z_3)$$

$$\underline{\underline{= 1 + 2\cos\left(\frac{2\pi}{5}\right) + 2\cos\left(\frac{4\pi}{5}\right)}}$$

$$\cancel{z_1 \cdot z_2 \cdot z_3 \cdot z_4 \cdot z_5 = \ldots}$$

$$z_1 z_2 z_3 z_4 z_5 = z$$

$$|z| = |z_1||z_2||z_3||z_4||z_5| = 1$$

$$\arg(z) = \sum_{n=1}^{5} \arg(z_n) = 4\pi \quad \to \quad \underline{\underline{z = 1}}$$