MATTE3 UKE 12

(1) ~~Beause~~ $\{b_1, b_2 \ldots b_n\}$ and $\{c_1, \ldots, c_n\}$ form bases, they must be linearly independent, sets.

$$\sum_j a_{ij} b_j = a_{i1} b_1 + a_{i2} b_2 \ldots + a_{in} b_n = c_i$$

if any two rows of A are linear combinations of each other, then

$$\sum_j a_{ij} b_j = x \sum a_{kj} b_j$$ for these two rows.

Thus, for there to exist n linearly independent vectors c, the rank of A must be n.

(2) $P(x) = a_0 + a_1 x + a_2 x^2$

Basis for P: $\{\vec{v}_0, \vec{v}_1, \vec{v}_2 \mid P(0)\vec{v}_0 + P(1)\vec{v}_1 + P(2)\vec{v}_2 = P(x)\}$

$P(0) = a_0$, $P(1) = a_0 + a_1 + a_2$, $P(2) = a_0 + 2a_1 + 4a_2$

$$a_0 \vec{v}_0 + (a_0 + a_1 + a_2)\vec{v}_1 + (a_0 + 2a_1 + 4a_2)\vec{v}_2 = a_0 + a_1 x + a_2 x^2$$

$$a_0(\vec{v}_0 + \vec{v}_1 + \vec{v}_2) + a_1(\vec{v}_1 + 2\vec{v}_2) + a_2(\vec{v}_1 + 4\vec{v}_2) = a_0 + a_1 x + a_2 x^2$$

I $\vec{v}_0 + \vec{v}_1 + \vec{v}_2 = 1$

II $0\vec{v}_0 \quad \vec{v}_1 + 2\vec{v}_2 = x$

III $0\vec{v}_0 \quad \vec{v}_1 + 4\vec{v}_2 = x^2$

row reduction $\rightarrow$

$\{\vec{v}_0, \vec{v}_1, \vec{v}_2\}$ form a basis for P_2

$\vec{v}_0 = 1 + \frac{1}{2}(x^2 - 3x)$, $\vec{v}_1 = 2x - x^2$, $\vec{v}_2 = \frac{1}{2}(x^2 - x)$

(2b)

$P(x) = a_0 + a_1 x + a_2 x^2$

$P(0) = a_0$, $P'(0) = a_1$, $P''(0) = 2a_2$

$a_0 \vec{V_0} + a_1 \vec{V_1} + 2a_2 \vec{V_2} = P(x)$

$\left\{ \vec{V_0} = 1 , \vec{V_1} = x , \vec{V_2} = \frac{x^2}{2} \right\}$ form a basis for P_2

(2c) $P(x) = x^2$

System 1) coordinates $= \begin{pmatrix} P(0) \\ P(1) \\ P(2) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$

System 2) coordinates $= \begin{pmatrix} P(0) \\ P'(0) \\ P''(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

(2d) $B = \left\{ 1 + \frac{1}{2}(x^2 - 3x), 2x - x^2, \frac{1}{2}(x^2 - x) \right\}$

$C = \left\{ 1, x, \frac{1}{2}x^2 \right\}$

$_C P_B = \left[[\vec{b_1}]_C \ [\vec{b_2}]_C \ [\vec{b_3}]_C \right] = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 2 & -\frac{1}{2} \\ 1 & -2 & 1 \end{bmatrix}$