

MATTE3 VKE 11

(1) Because $\{b_1, b_2, \dots, b_n\}$ and $\{c_1, \dots, c_n\}$ form bases, they must be linearly independent sets.

$$\sum_i a_{ij} b_j = a_{i1} b_1 + a_{i2} b_2 + \dots + a_{in} b_n = c_i$$

if any two rows of A are linear combinations of each other, then

$$\sum_i a_{ij} b_j = x \sum_i a_{ik} b_k \quad \text{for these two rows.}$$

Thus, for there to exist n linearly independent vectors c , the rank of A must be n .

(2) $P(x) = a_0 + a_1 x + a_2 x^2$

basis for P : $\{\vec{v}_0, \vec{v}_1, \vec{v}_2 \mid P(0)\vec{v}_0 + P(1)\vec{v}_1 + P(2)\vec{v}_2 = P(x)\}$

$P(0) = a_0$, $P(1) = a_0 + a_1 + a_2$, $P(2) = a_0 + 2a_1 + 4a_2$

$$a_0 \vec{v}_0 + (a_0 + a_1 + a_2) \vec{v}_1 + (a_0 + 2a_1 + 4a_2) \vec{v}_2 = a_0 + a_1 x + a_2 x^2$$

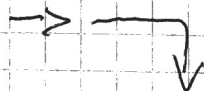
$$a_0(\vec{v}_0 + \vec{v}_1 + \vec{v}_2) + a_1(\vec{v}_1 + 2\vec{v}_2) + a_2(2\vec{v}_1 + 4\vec{v}_2) = a_0 + a_1 x + a_2 x^2$$

I $\vec{v}_0 + \vec{v}_1 + \vec{v}_2 = 1$

II $\vec{v}_0 + \vec{v}_1 + 2\vec{v}_2 = x$

III $\vec{v}_0 + \vec{v}_1 + 4\vec{v}_2 = x^2$

Row reduction



~~$\vec{v}_0 = 1 - x + x^2$~~
 ~~$\vec{v}_1 = x - x^2$~~
 ~~$\vec{v}_2 = x^2 - x$~~

$\{\vec{v}_0, \vec{v}_1, \vec{v}_2\}$ form a basis for P_2

$\vec{v}_0 = 1 + \frac{1}{2}(x^2 - 3x)$, $\vec{v}_1 = 2x - x^2$, $\vec{v}_2 = \frac{1}{2}(x^2 - x)$

CZB)

$$P(x) = a_0 + a_1x + a_2x^2$$

$$P(0) = a_0, \quad P'(0) = a_1, \quad P''(0) = 2a_2$$

$$a_0\vec{v}_0 + a_1\vec{v}_1 + 2a_2\vec{v}_2 = P(x)$$

$\{\vec{v}_0 = 1, \vec{v}_1 = x, \vec{v}_2 = \frac{x^2}{2}\}$ form a basis for P_2

CZC)

$$P(x) = x^2$$

System 1)

$$\text{coordinates} = \begin{pmatrix} P(0) \\ P(1) \\ P(2) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$$

$$\text{System 2) coordinates} = \begin{pmatrix} P(0) \\ P'(0) \\ P''(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

CZd)

$$B = \left\{ 1 + \frac{1}{2}(x^2 - 3x), 2x - x^2, \frac{1}{2}(x^2 - x) \right\}$$

$$C = \left\{ 1, x, \frac{1}{2}x^2 \right\}$$

$${}_C P_B = \begin{bmatrix} [B_1]_C & [B_2]_C & [B_3]_C \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 2 & -\frac{1}{2} \\ 1 & -2 & 1 \end{bmatrix}$$