C1) ① $N=1$

$$F(a)+F(b) = a_0 + a_1\sin(x) + b_0 + b_1\sin(x)$$
$$F(a+b) = (a_0+b_0) + (a_1+b_1)\sin(x)$$

$$F(a), F(b) \in V \rightarrow F(a+b) \in V$$

② $F(a)+(F(b)+F(c)) = F(a)+F(b+c)$, as shown in ①

$= F(a+b+c)$, $\forall\ a,b,c \in \mathbb{C}$

$= F(a+b)+F(c)$, as shown in ①

$$(F(a)+F(b))+F(c) = (F(a)+F(b))+F(c)$$

③

$$F(a)+F(0) = a_0 + a_1\sin(x) + 0_0 + 0_1\sin(x)$$
$$F(a)+F(0) = F(a)$$

④

$$F(a) + (-F(a)) = 0$$
$$a_0 + a_1\sin(x) - a_0 - a_1\sin(x) = 0$$

⑤

$$rF(a) = ra_0 + ra_1\sin(x) = F(ra) \in V$$

⑥

$$r(F(a)+F(b)) = ra_0 + ra_1\sin(x) + rb_0 + rb_1\sin(x)$$
$$= rF(a) + rF(b)$$

⑦

$$(c+d)F(a) = (c+d)a_0 + (c+d)a_1\sin(x)$$
$$= ca_0 + ca_1\sin(x) + da_0 + da_1\sin(x)$$
$$(c+d)F(a) = cF(a) + dF(a)$$

(8) $c(df(a)) = cf(da)$, shown in (6)

$= f(cda)$ $\forall c, d \in \mathbb{C}$

$= f(dca)$

$= df(ca)$

$c(df(a)) = d(cf(a))$

(9) $1 \cdot f(a) = 1 \cdot a_0 + 1 \cdot a_1 \sin(x)$

$= f(a)$

C2.

$\det(A) \neq 0 \iff A$ is invertible

A is invertible $\iff$ columns of A are linearly independent

[n linearly independent columns span $\mathbb{R}^n$]