

**Remark:** Hand in at least two of the problems listed under **C**.

# A - Reading

- $L \equiv Lay$  (the part of the textbook that deals with Linear Algebra)
- L 5.1 Eigenvectors and Eigenvalues
- L 5.2 The Characteristic Equation
- L 5.3 Diagonalization
- L 5.5 Complex Eigenvalues

# **B** - Finger Exercises

## **B.1**

Find the eigenvalues and the corresponding eigenvectors for the matrix  $\begin{bmatrix} -2 & 2 \\ -2 & 3 \end{bmatrix}$ .

## **B.2**

Find the eigenvalues and the corresponding eigenvectors for the matrix  $\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$ .

## **B.3**

Let

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Find an invertible matrix S and a diagonal matrix D such that  $A = SDS^{-1}$ . Compute  $A^{30}$ .

## **B.4**

Find a basis of complex 3-vectors and complex eigenvalues for the matrix

0	0	1	
1	0	0	
0	1	0	

# B.5

Find a basis of  $\mathbb{R}^3$  that consist of eigenvectors of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}.$$

# C - Exam Preparation

C.1

a) Find the characteristic polynomial of the matrix

$$A_5 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

(Optional) Generalize the previous result to a general  $n \times n$  matrix of the form of  $A_5$ .

C.2

Assume that  $a \neq d$ . Find an invertible matrix S such that

$$S\begin{bmatrix}a&b\\0&d\end{bmatrix}S^{-1} = \begin{bmatrix}a&0\\0&d\end{bmatrix}.$$

# **C.3**

Let  $p(\lambda) = \lambda^2 + a\lambda + b$  be a quadratic polynomial. Find a 2×2-matrix with characteristic polynomial p.

# D - Relevant Exercises From the Book

The following roughly divides the exercises from the relevant book chapters into two categories. Note that the book contains answers to odd-numbered exercises.

## Exercises to train computing skills:

- Section 5.1, Problems 1-18
- Section 5.2, Problems 1-17
- Section 5.3, Problems 1-20
- Section 5.5, Problems 1-20

## Exercises that require understanding, thought and maybe a good idea ;)

• Section 5.1, Problems 19-35

- Section 5.2, Problems 18-27
- $\bullet$  Section 5.3, Problems 21-32
- $\bullet$  Section 5.5, Problems 21-26