

TMA4115 Calculus 3 Spring 2018

Work Sheet Week 5

Remark: Hand in at least two of the problems listed under **C**.

A - Reading

- $L \equiv Lay$ (the part of the textbook that deals with Linear Algebra)
- L 1.3 Vector Equations
- L 1.4 The Matrix Equation $A\mathbf{x} = \mathbf{b}$
- L1.5 Solution Sets of Linear Systems

B - Finger Exercises

B.1

Write the system

of linear equations in matrix form. Find the general solution of it in parametric from. Can you interpret the result geometrically? Can you make a picture?

B.2

Let

$$\mathbf{v} = \begin{bmatrix} 1\\0\\-2 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} -3\\\mathbf{i}\\8 \end{bmatrix}, \text{ and } \mathbf{y} = \begin{bmatrix} h\\-5\mathbf{i}\\-3 \end{bmatrix}.$$

For what value(s) of h is the vector \mathbf{y} in the plane generated by the vectors \mathbf{v} and \mathbf{w} ?

B.3

Let

$$\mathbf{u} = \begin{bmatrix} 0\\0\\-2\mathbf{i} \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 0\\-3\\8\mathbf{i} \end{bmatrix}, \text{ and } \mathbf{w} = \begin{bmatrix} 4\\-1\\-5 \end{bmatrix}.$$

Does the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ of vectors span \mathbb{C}^3 ?

C - Exam Preparation

C.1

Let

$$A = \begin{bmatrix} a & 1 \\ 1 & a \end{bmatrix}$$
 and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.

Find all **x** and λ (in terms of *a*) such that the equation A**x** = λ **x** holds.

C.2

Consider the vectors

$$\mathbf{v} = \begin{bmatrix} i \\ i \\ i \end{bmatrix}$$
 and $\mathbf{w} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

Find all vectors \mathbf{u} such that the set $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ of vectors spans \mathbb{C}^3 .

C.3

Suppose that the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution. Explain that the solution is unique if and only if the equation $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $(\mathbf{x} = \mathbf{0})$.

Extra: Can you prove the statement in C.3?