Innlevering 4

Oppgave 1)

Flaten $z^2 = x^2 + y^2$ mellom $z=0$ og $x+2z=3$

$$\text{Areal} = \iint_S dS = \iint_D \left| \frac{\partial \vec{r}}{\partial x} \times \frac{\partial \vec{r}}{\partial y} \right| dx\,dy$$

$$\vec{r} = \left[x, y, \sqrt{x^2+y^2}\right]$$

$$\frac{\partial \vec{r}}{\partial x} = \left[1, 0, \frac{x}{\sqrt{x^2+y^2}}\right], \quad \frac{\partial \vec{r}}{\partial y} = \left[0, 1, \frac{y}{\sqrt{x^2+y^2}}\right]$$

$$\frac{\partial \vec{r}}{\partial x} \times \frac{\partial \vec{r}}{\partial y} = \left[\frac{-x}{\sqrt{x^2+y^2}}, \frac{-y}{\sqrt{x^2+y^2}}, 1\right]$$

$$\text{Areal} = \iint_D \sqrt{\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} + 1}\; dx\,dy$$

$$= \iint_D \sqrt{2}\, dx\,dy = \iint_P \sqrt{2}\, r\, dr\, d\theta$$

$$\text{Areal} = \iint_D \sqrt{2}\, dx\,dy$$

$$x - 2z = 3 \rightarrow \underline{z = \frac{x-3}{2}}$$

$$z^2 = x^2 + y^2 \rightarrow \left(\frac{x-3}{2}\right)^2 = x^2 + y^2$$

$$x^2 - 6x + 9 = 4x^2 + 4y^2$$

$$3x^2 + 6x + 4y^2 = 9$$

$$x^2 + 2x + \frac{4}{3}y^2 = 3$$

$$(x+1)^2 - 1 + \left(\frac{2}{\sqrt{3}}y\right)^2 = 3$$

$$(x+1)^2 + \left(\frac{2}{\sqrt{3}}y\right)^2 = 4$$

$$u^2 + v^2 = 4$$

$$u = x + 1 \rightarrow x = u - 1$$

$$v = \frac{2}{\sqrt{3}}y \rightarrow y = \frac{\sqrt{3}}{2}v$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 1 & 0 \\ 0 & \frac{\sqrt{3}}{2} \end{vmatrix} = \underline{\frac{\sqrt{3}}{2}}$$

$$\iint_D \sqrt{2}\, dx\,dy = \iint_R \sqrt{2} \cdot \frac{\sqrt{3}}{2}\, du\,dv$$

Hvor R er avgrenset av $u^2 + v^2 = 4$

$$\iint_R \frac{\sqrt{6}}{2}\, du\,dv = \int_0^{2\pi} \int_0^2 \frac{\sqrt{6}}{2} r\, dr\, d\theta$$

$$= 2\pi \cdot \frac{\sqrt{6}}{2} \cdot \frac{1}{2} \cdot 2^2$$

$$\underline{\underline{= 2\pi\sqrt{6}}}$$

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Oppgave 2) $I = \oint_C (xy + \ln(x^2+1))dx + (4x + e^{y^2} + 3\arctan y)dy$

$$\int_{\partial D} P\,dx + Q\,dy = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA$$

$\underline{C = \partial D}$, $\underline{Q = 4x + e^{y^2} + 3\arctan y}$, $\underline{P = xy + \ln(x^2+1)}$

$\underline{\frac{\partial Q}{\partial x} = 4}$, $\underline{\frac{\partial P}{\partial y} = x}$

$$I = \iint_D 4 - x\,dA \qquad D = \{(x,y) \mid x^2+y^2 \le 1,\ y \ge 0\}$$

$$D = \{(r,\theta) \mid r \le 1,\ 0 \le \theta \le \pi\}$$

$$= \int_0^\pi \int_0^1 (4 - r\cos\theta)\,r\,dr\,d\theta$$

$$= \int_0^\pi \frac{4}{2} - \frac{1}{3}\cos\theta\,d\theta = 2\pi - \frac{1}{3}\left[\sin\theta\right]_0^\pi$$

$$\underline{\underline{I = 2\pi}}$$

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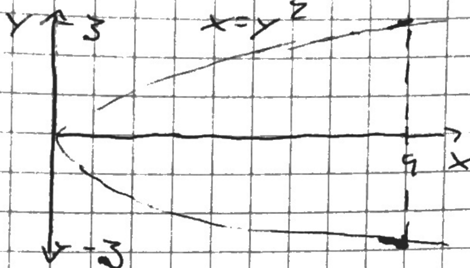
Oppgave 3)

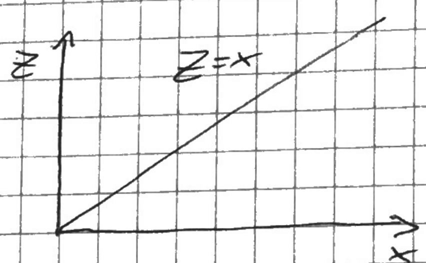
$x = y^2 \quad , \quad x = 9 \quad , \quad z = 0 \quad , \quad x = z$ avgrenser T

$$F(x,y,z) = [3x - 5y,\ 4z - 2y,\ 8yz]$$

$$I = \iint_{\partial T} \vec{F} \cdot \vec{n}\, dS = \iiint_{T} \vec{\nabla} \cdot \vec{F}\, dV$$

$$\underline{\vec{\nabla} \cdot \vec{F} = 3 - 2 + 8y = 1 + 8y}$$





$$I = \int_{-3}^{3} \int_{y^2}^{9} \int_{0}^{x} 1 + 8y \, dz\, dx\, dy$$

$$I = \int_{-3}^{3} \int_{y^2}^{9} (1 + 8y)x \, dx\, dy$$

$$I = \frac{1}{2} \int_{-3}^{3} (1 + 8y)(9^2 - y^2)\, dy = \frac{1}{2} \int_{-3}^{3} 9^2 + 8y^3 + 8 \cdot 9^2 y - y^2 \, dy$$

$$\boxed{k \int_{-a}^{a} y^{2n+1}\, dy = 0 \text{, gitt at } n \in \mathbb{N}} \quad (1)$$

$$I = \frac{1}{2} \int_{-3}^{3} 9^2 - y^2 \, dy \quad (1)$$

$$= 3 \cdot 9^2 - \frac{1}{6}\left[3^3 - (-3)^3\right]$$

$$= 3 \cdot 9^2 - \frac{1}{6}(2 \cdot 3 \cdot 9)$$

$$\underline{\underline{I = 234}}$$

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Oppgave 4)

$$\vec{F}(x,y,z)=[x+y,\,4x-y,\,z^2+xy]$$

$$\vec{\nabla}\cdot\vec{F}=1-1+2z=2z$$

$\underline{\vec{\nabla}\cdot\vec{F}=2z}$ $\rightarrow$ $\underline{\vec{F} \text{ er ikke konservativt}}$

$$\vec{\nabla}\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x+y & 4x-y & z^2-xy\end{vmatrix}=[-x,\,y,\,3]$$

$$\underline{\vec{\nabla}\times\vec{F}=[-x,\,y,\,3]}$$

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Oppgave 4B) $F(x,y,z) = [x+y, 4x-y, z^2+xy]$

$z = \sqrt{x^2+y^2+1}$ $\qquad z = \sqrt{10}$

$\vec{r}(t) = [-3\sin t, 3\cos t, \sqrt{10}]$, skjæringskurve

$$\oint_C \vec{F}\, d\vec{r} = \iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, dS$$

$$d\vec{r} = \frac{d\vec{r}}{dt}\, dt = [-3\cos(t), -3\sin(t), 0]\, dt$$

Fordi S er ~~parallell~~ parallell med $z=0$.

Blir $\underline{\vec{n} = [0,0,1]}$

$$\iint_S [-x, y, 3] \cdot [0,0,1]\, dS \quad , \quad \underline{S = \{(x,y,z) \mid x^2+y^2 \le 3^2,\ z = \sqrt{10}\}}$$

$$= \int_0^{2\pi} \int_0^3 3r\, dr\, d\theta$$

$$\underline{\underline{= 6\pi \cdot \frac{9}{2} = 27\pi}}$$