Innlevering 3

Oppg. 1)

$$m = \iiint_T \rho \, dV$$

$\rho = 1$

$z = 4 - x^2 - y^2 \xrightarrow{\text{Sylinder}} z = 4 - r^2$

$dx\,dy\,dz = r\,dr\,d\theta\,dz$

$\underline{r = \sqrt{4-z}}$

$\underline{r = 0 \rightarrow z = 4}$

$$m = \int_0^{2\pi} \int_3^4 \int_0^{\sqrt{4-z}} r \, dr \, dz \, d\theta$$

$$m = 2\pi \int_3^4 \frac{1}{2}\left[r^2\right]_0^{\sqrt{4-z}} dz$$

$$m = 2\pi \cdot \frac{1}{2} \int_3^4 4 - z \, dz$$

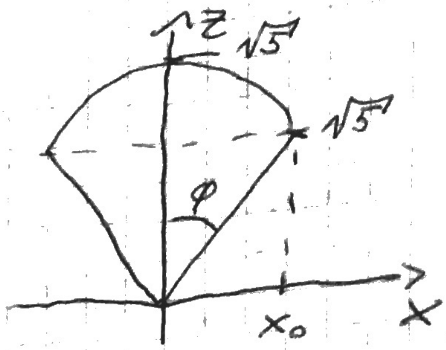
$$m = \pi \left[4z - \frac{1}{2}z^2\right]_3^4$$

$$m = \pi\left(8 - \left(12 - \frac{9}{2}\right)\right)$$

$$\underline{\underline{m = \frac{\pi}{2}}}$$

Innlevering 3

Oppg 2)



$$z = 2\sqrt{x^2+y^2}$$
$$x^2+y^2+z^2 = 5$$

Omgjøring til kulekoordinater:

$\underline{0 \le r \le \sqrt{5}}$

$\underline{0 \le \varphi \le \arccos\left(\frac{2}{\sqrt{5}}\right)}$

$$z = 2\sqrt{x_0^2 + 0^2}$$
$$\underline{z = 2x_0}$$
$$x_0^2 + 0^2 + (2x_0)^2 = 5$$
$$\underline{x_0 = 1 \to z = 2}$$

$0 \le \theta \le 2\pi$

$$I = \iiint_T \exp(x^2+y^2+z^2)^{3/2}\, dV = \int_0^{2\pi}\int_0^{\arccos\left(\frac{2}{\sqrt{5}}\right)}\int_0^{\sqrt{5}} \exp(r^2)^{3/2} \cdot r^2 \sin\varphi \, dr\, d\varphi\, d\theta$$

$$I = 2\pi \int_0^{\arccos\left(\frac{2}{\sqrt{5}}\right)}\int_0^{\sqrt{5}} r^2 e^{r^3} \sin\varphi \, dr\, d\varphi = \frac{2\pi}{3}\int_0^{\arccos\left(\frac{2}{\sqrt{5}}\right)} \left(e^{5^{3/2}} - 1\right)\sin\varphi\, d\varphi$$

$$I = \frac{2\pi\left(e^{5\sqrt{5}} - 1\right)}{3}\Big[-\cos\varphi\Big]_0^{\arccos\left(\frac{2}{\sqrt{5}}\right)}$$

$$I = \frac{2\pi}{3}\left(e^{5\sqrt{5}} - 1\right)\left(-\frac{2}{\sqrt{5}} - (-1)\right)$$

$$\underline{\underline{I = \frac{2\pi}{3}\left(e^{5\sqrt{5}} - 1\right)\left(1 - \frac{2}{\sqrt{5}}\right)}}$$

Innlevering 3

Oppgave 3)

$f(x,y,z) = 8\sqrt{x} + 2y^2$

$r(t) = [t^2, e^t, t]$

$$P = \int f(x,y,z)\,ds = \int f(r(t))\,|v(t)|\,dt$$

$$P = \int_0^1 (8t + 2e^{2t})\,|[2t, e^t, 1]|\,dt$$

$$P = \int_0^1 (8t + 2e^{2t})\sqrt{4t^2 + e^{2t} + 1}\,dt$$

$$P = \int_{t=0}^{t=1} \frac{8t + 2e^{2t}}{8t + 2e^{2t}}\sqrt{u}\,du \quad , \; t=0 \rightarrow u = (\sqrt{2})^2$$

$$t = 1 \rightarrow u = (\sqrt{e^2+5})^2$$

$$P = \int_2^{e^2+5} \sqrt{u}\,du = \left[\frac{2}{3}u^{3/2}\right]_2^{e^2+5}$$

$$P = \frac{2}{3}\left((e^2+5)^{3/2} - 2\sqrt{2}\right)$$

$$\vec{F}(x,y,z) = \left(z e^{xz+y},\ e^{xz+y} + 2z,\ x e^{xz+y} + 2y\right)$$

Oppg 4) $\vec{F}$ er konservativt hvis det eksisterer en $f(x,y,z) \mid \nabla f = \vec{F}$

$$\frac{\partial f}{\partial x} = z e^{xz+y},\quad \frac{\partial f}{\partial y} = e^{xz+y} + 2z,\quad \frac{\partial f}{\partial z} = x e^{xz+y} + 2y$$

$$f = e^{xz+y} + C(z,y),\quad f = e^{xz+y} + C(x,z),\quad f = e^{xz+y} + C(x,y)$$

$$\frac{\partial f}{\partial y} = e^{xz+y} + C(z) = e^{xz+y} + 2z$$

$$\underline{C(z) = 2z}$$

$$\frac{\partial f}{\partial z} = x e^{xz+y} + C(y) = x e^{xz+y} + 2y$$

$$\underline{C(y) = 2y}$$

$$\underline{f(x,y,z) = e^{xz+y} + 2yz}$$

Fortsettelse Oppg. 4)

Fordi $f(x,y,z) = e^{xz+y} + zyz$

gir $\nabla f = \vec{F}$ er $\vec{F}(x,y,z)$ konservativt.

$$\int_0^{\pi/2} \vec{F}\, d\vec{r} = \vec{F}(\vec{r}(\pi/2)) - \vec{F}(\vec{r}(0))$$, fordi $\vec{F}$ er konservativt

$\vec{r}(t) = [\cos(t), \sin(t), t]$

$\vec{r}(0) = (1, 0, 0)$

$\vec{r}(\pi/2) = (0, 1, \pi/2)$

$-\vec{F}(1,0,0) + \vec{F}(0,1,\pi/2) = -(e^0 + 0) + (e + \pi)$

$$\underline{\underline{\int_0^{\pi/2} \vec{F}\, d\vec{r} = -1 + e + \pi}}$$