Innlevering 2

Oppgave 1)

$$f(x,y) = \frac{1-2xy}{x^2+y^2} \qquad (x,y) \neq (0,0)$$

$\lim_{(x,y)\to(0,0)} f(x,y) = \left[\frac{1}{0}\right]$, $f(x,y)$ vokser ubegrenset når $(x,y) \to (0,0)$, altså f har ingen største verdi

$$f(x,y) = \frac{1}{x^2+y^2} - \frac{2xy}{x^2+y^2}$$

$x^2 \geq xy$, Hvis $x \geq y$ $\quad \to x^2+y^2 \geq 2xy \quad \forall (x,y) \in \mathbb{R}$

$y^2 \geq xy$, Hvis $y \geq x$

$x^2+y^2 = 2xy \quad \to \quad x = y$

den minste verdien til $f(x,y)$ finnes når $\frac{2xy}{x^2+y^2}$ er størst , på linjen $x = y$

$f(x,x) = \frac{1}{2x^2} - 1$ vokser asymtotisk mot $f = -1$

f har derfor ingen minste verdi

Oppgave 2)

$F(x,y,z) = z$ $\qquad x^2+2y^2=1$ $\qquad z = x-4y$

$x = \pm\sqrt{1-2y^2}$ $\qquad \underline{z = \pm\sqrt{1-2y^2}-4y}$

Største verdien til z langs skjæringskurven er ved maks til $\pm\sqrt{1-2y^2}-4y$

$g(y) := \pm\sqrt{1-2y^2}-4y$

$$g'(y) = \frac{\pm 4y}{2\sqrt{1-2y^2}} - 4 = 0$$

$$\pm y = 2\sqrt{1-2y^2}$$

$$\frac{1}{4}y^2 = 1-2y^2$$

$$y^2 = \frac{4}{9}$$

$$\underline{y = \pm\frac{2}{3}}$$

$g\left(\frac{2}{3}\right) = \frac{1}{3} - \frac{8}{3} = -\frac{7}{3}$ $\quad \vee \quad$ $\underline{g\left(\frac{2}{3}\right) = -\frac{1}{3} - \frac{8}{3} = -3}$

$g\left(-\frac{2}{3}\right) = -\frac{1}{3} + \frac{8}{3} = \frac{7}{3}$ $\quad \vee \quad$ $\underline{g\left(-\frac{2}{3}\right) = \frac{1}{3} + \frac{8}{3} = 3}$

Største verdi til $F(x,y,z)$ langs kurven er 3, minste verdi er -3

Innlevering 2

Oppgave 3)

$$\iint_R \sin\left(\frac{x-y}{x+y}\right) dA$$ Hvor R er trapeset med hjørner $(1,1),(2,2)(4,0)$ og $(2,0)$

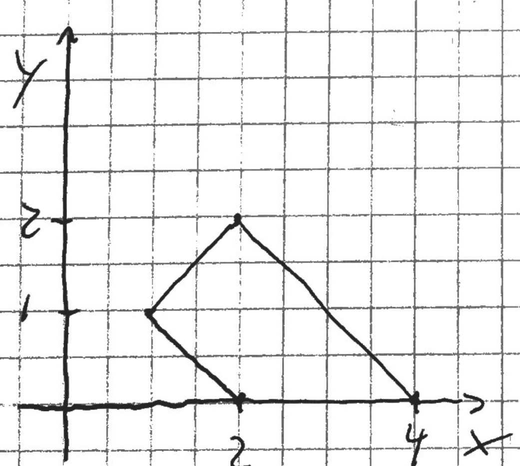
$u=x-y$, $v=x+y$ er lineær substitusjon som gir et trapes i uv-planet med hjørner

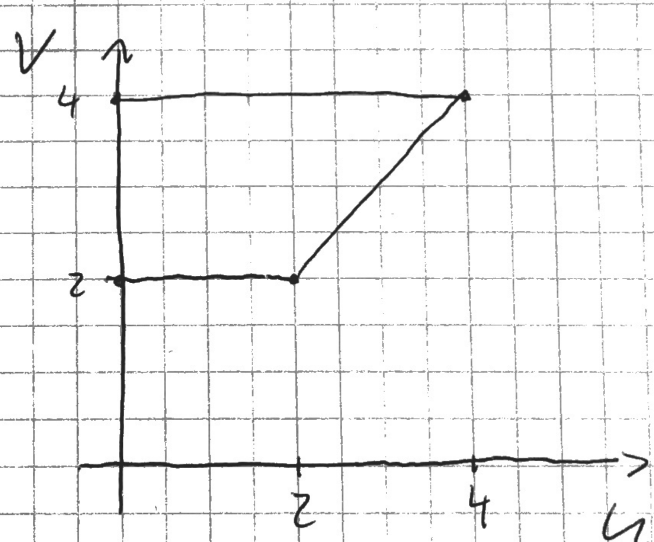
$(0,2)$, $(0,4)$, $(4,4)$ og $(2,2)$. $\left|\frac{\partial(x,y)}{\partial(u,v)}\right| = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2}$

$$\int_2^4\int_0^v \sin\left(\frac{u}{v}\right)\cdot\frac{1}{2}\,du\,dv = \int_2^4\left[-\frac{1}{2}v\cos\left(\frac{u}{v}\right)\right]_0^v dv$$

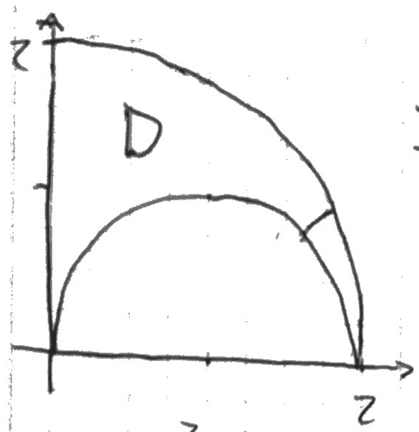
$$= -\frac{1}{2}\int_2^4 v(\cos(1)-1)\,dv = -\frac{1}{4}(\cos(1)-1)\left[v^2\right]_2^4$$

$$\underline{\underline{= -3(\cos(1)-1)}}$$





Oppgave 4)



$$I = \int_0^2 \int_{\sqrt{2x-x^2}}^{\sqrt{4-x^2}} \sqrt{4-x^2-y^2}\, dy\, dx$$

$$I = \int_0^2 \int_0^{\sqrt{4-x^2}} \sqrt{4-x^2-y^2}\, dy\, dx - \int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{4-x^2-y^2}\, dy\, dx$$

$x = r\cos\theta$, $y = r\sin\theta$

$y = \sqrt{2x - x^2} \;\; \rightarrow \;\; \underline{r = 2\cos\theta}$

$y = \sqrt{4 - x^2} \;\; \rightarrow \;\; \underline{r = 2}$

$dy\,dx = r\,dr\,d\theta$

$$I = \int_0^{\pi/2} \int_0^2 \sqrt{4-r^2}\, r\, dr\, d\theta - \int_0^{\pi/2} \int_0^{2\cos\theta} \sqrt{4-r^2}\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \frac{1}{3} \left[(4-r^2)^{3/2} \right]_0^2 d\theta - \int_0^{\pi/2} \frac{1}{3} \left[(4-r^2)^{3/2} \right]_0^{2\cos\theta} d\theta$$

$$= \int_0^{\pi/2} \frac{4^{3/2}}{3}\, d\theta - \int_0^{\pi/2} \frac{1}{3} \left((4 - 4\cos^2\theta)^{3/2} - 4^{3/2} \right) d\theta$$

bruker delvis integrasjon
$u' = \sin\theta$
$v = \sin^2\theta$

$$= \frac{2 \cdot 4^{3/2}}{3} \int_0^{\pi/2} d\theta - \int_0 \frac{4^{3/2}}{3} \sin^3\theta\, d\theta$$

$$= \frac{8\pi}{3} - \frac{4^{3/2}}{3} \left(\left[-\cos\theta \sin^2\theta \right]_0^{\pi/2} - \int_0^{\pi/2} -\cos\theta \cdot 2\sin\theta\cos\theta\, d\theta \right)$$

$$= \frac{8\pi}{3} - \frac{4^{3/2}}{3} \left(0 + \int_0^{\pi/2} \sin(2\theta)\cos\theta\, d\theta \right)$$

$t = \sin(2\theta)$
$d\theta = \frac{dt}{2\cos\theta}$

$$= \frac{8\pi}{3} - \frac{8}{3} \cdot \frac{1}{4} \left[\sin^2(2\theta) \right]_0^{\pi/2}$$

$$= \underline{\underline{\frac{8\pi}{3}}}$$