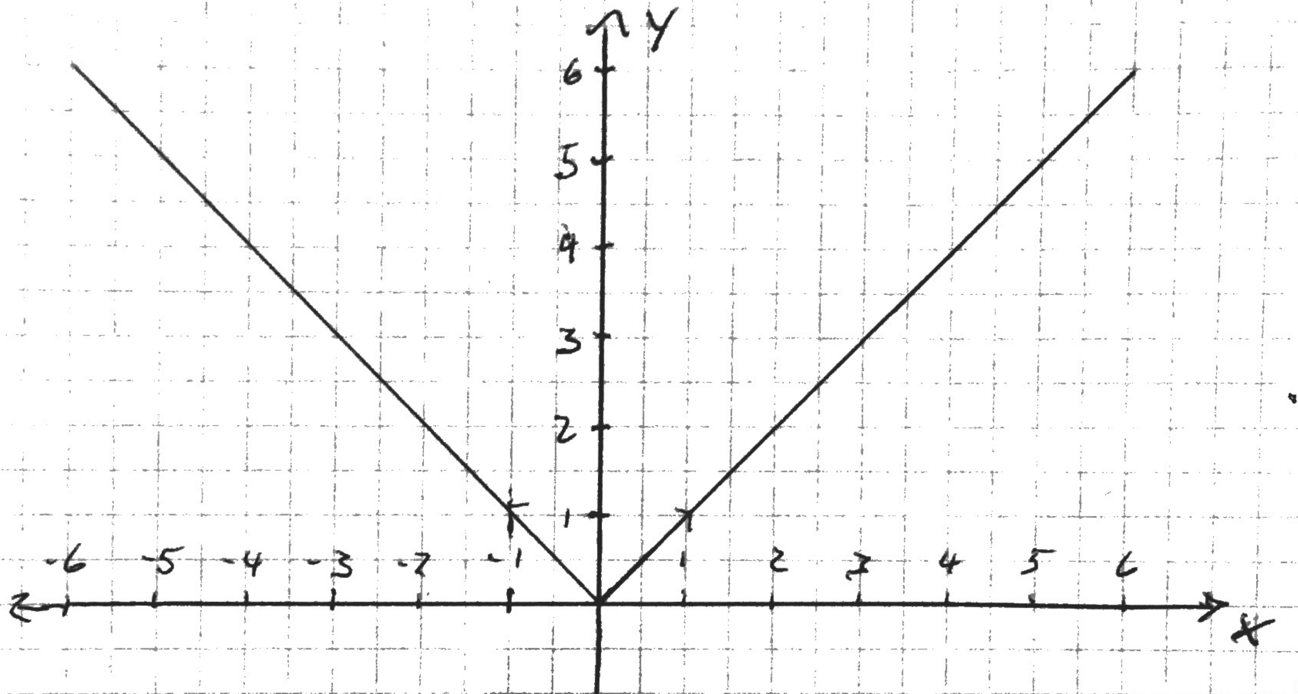
Innlevering 1 Matte 2

Oppgave 1

$x(t)=t^5$, $y(t)=t^4|t|$

$t = x^{1/5}$ $\quad y = x^{4/5}\cdot|x^{1/5}|$

$y = |x|$



$\lim_{x\to 0^-} y'(x) = -1$

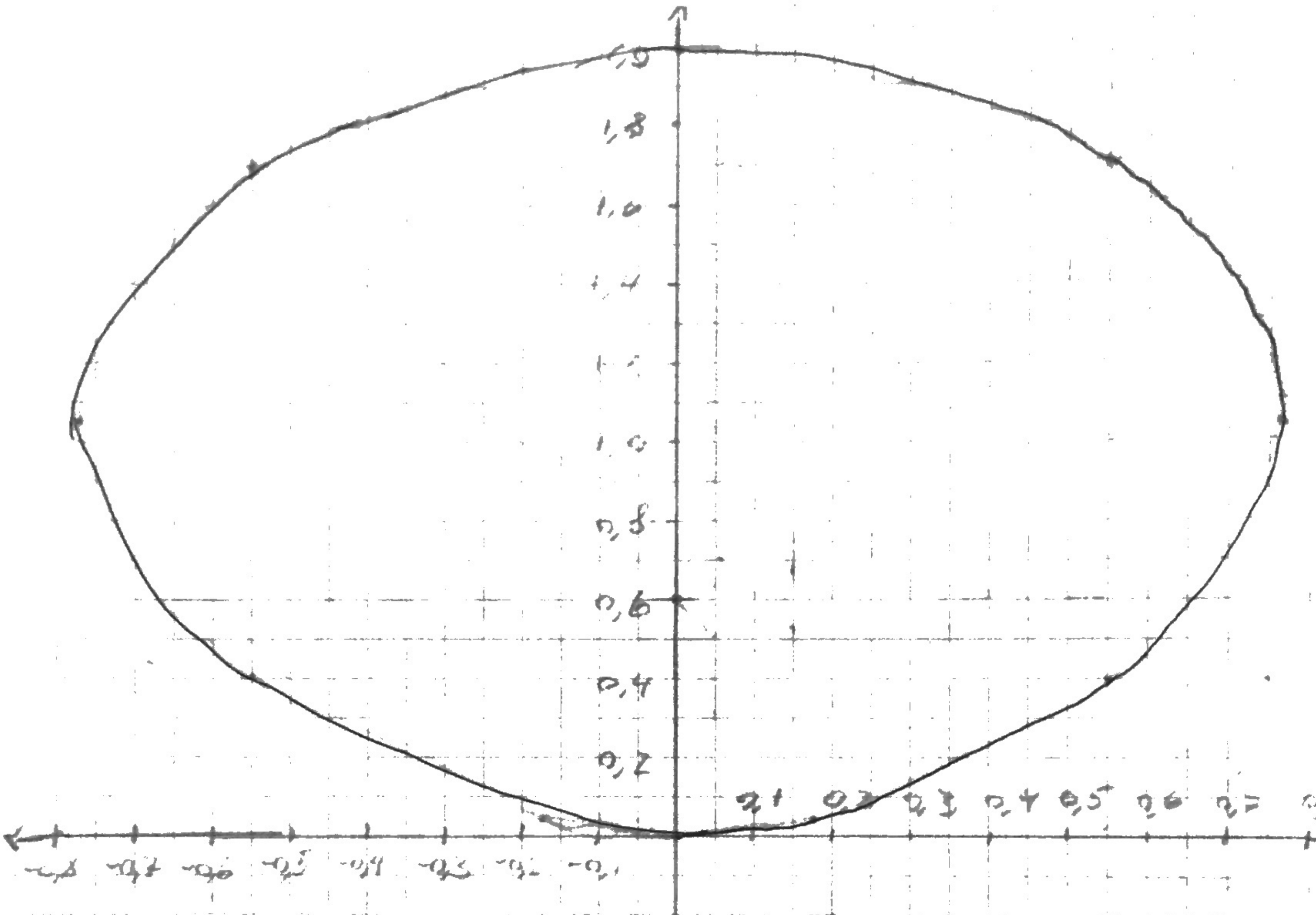
$\lim_{x\to 0^+} y'(x) = 1$

$y'(x)$ er ikke kont. -> kurven er ikke glatt

Oppgave 2)

$$r = 1-\cos(2\theta) \quad , \quad 0 \leq \theta \leq \pi$$

$$x(\theta) = r\cos\theta = \cos\theta(1-\cos(2\theta))$$
$$y(\theta) = r\sin\theta = \sin\theta(1-\cos(2\theta))$$



$$A = \frac{1}{2}\int_{\theta_1}^{\theta_2} r^2\,d\theta - \frac{1}{2}\int_{\theta_1}^{\theta_2} 1^2\,d\theta$$

$$A = \frac{1}{2}\int_{\pi/4}^{3\pi/4} (1-\cos(2\theta))^2 - 1\,d\theta$$

$$A = \frac{1}{2}\int_{\pi/4}^{3\pi/4} 1 - 2\cos(2\theta) + \cos^2(2\theta) - 1\,d\theta$$

$$A = \frac{1}{2}\int_{\pi/4}^{3\pi/4} \cos^2(2\theta) - 2\cos(2\theta)\,d\theta$$

$$\cancel{A = \frac{1}{2}\left[-4\sin(2\theta)\cos(2\theta) - 4\sin(2\theta)\right]_{\pi/4}^{3\pi/4}} =$$

$$A = \frac{1}{2}\int_{\pi/4}^{3\pi/4} \frac{\cos(4\theta)+1}{2} - 2\cos(2\theta)\,d\theta = \underline{\underline{\frac{\pi}{8} + 1}}$$

Skjæring mellom kurven og $r=1$:

$$1-\cos(2\theta) = 1$$
$$\cos(2\theta) = 0$$
$$2\theta = \frac{\pi}{2} + n\pi$$
$$\theta = \frac{\pi}{4} + n\frac{\pi}{2}$$

$\underline{\theta_1 = \frac{\pi}{4}}$ $\quad$ $\underline{\theta_2 = \frac{3\pi}{4}}$

Oppgave 3

$$\lim_{(x,y)\to(0,0)} \frac{\sin(2x)-2x+y}{x^3+y} = \lim_{x\to 0} \frac{\sin(2x)-2x}{x^3} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{\sin(2x)-2x}{x^3} = \lim_{x\to 0} \frac{2\cos(2x)-2}{3x^2} = \lim_{x\to 0} \frac{-4\sin(2x)}{6x}$$

$$= \lim_{x\to 0} \frac{-8\cos(2x)}{6} = \underline{-\frac{4}{3}}$$

$$\lim_{(x,y)\to(0,0)} \frac{\sin(2x)-2x+y}{x^3+y} = \lim_{y\to 0} \frac{y}{y} = \underline{1}$$

Fordi grenseverdien når du kommer langs x-aksen er forskjellig fra grensen når du kommer langs y-aksen eksisterer grensen <u>ikke</u>

Oppgave 4)

$$f(x,y) = \cos^2(x) + \sin(x)\cos(y)$$

$$\nabla f(x,y) = \left[-\sin(2x) + \cos(x)\cos(y),\ -\sin(y)\sin(x)\right]$$

$$L(x_1,y_1) = f(x_1,y_1) + \nabla f(x_1,y_1)\cdot(x-x_1,\ y-y_1)$$

$$L\left(\frac{\pi}{4},\frac{\pi}{4}\right) = 1 + \left[-\frac{1}{2},\ -\frac{1}{2}\right]\cdot\left[x-\frac{\pi}{4},\ y-\frac{\pi}{2}\right]$$

$$L\left(\frac{\pi}{4},\frac{\pi}{4}\right) = \frac{4+\pi}{4} - \frac{1}{2}(x+y)$$