Innlevering 4

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Oppgave 1a)

$$F(x) = \int_0^x e^{-t^2}\,dt$$

$$F'(x) = e^{-x^2}$$

Taylorrekken til $e^{-u} = \sum_{n=0}^{\infty} (-1)^n \frac{u^n}{n!}$

Taylorrekken til e^{-x^2}:

$$e^{-x^2} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$$

rekken konvergerer for alle $x \in \mathbb{R}$

$$F(x) = \int \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}\,dx$$

rekken konvergerer for alle $x \in \mathbb{R}$

$$f(x) = \int \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!} \, dx$$

$$F(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \frac{1}{2n+1} x^{2n+1}$$

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{n!(2n+1)}$$

1b) $|R_n(x)| \leq |a_{n+1}(x)|$ for en alternerende rekke.

$$0{,}0005 \leq \left| (-1)^{n+1} \cdot \frac{1^{2n+2}}{(n+1)!(2n+2)} \right|$$

$$0{,}0005 \leq \frac{1}{(n+1)!(2n+2)}$$

$$(n+1)!(2n+2) \geq 2000$$

prøver $n = 5$ $\rightarrow$ $6!(12) = 8640 > 2000$

$n = 4$ $\rightarrow$ $5!(10) = 1200 < 2000$

med 6 ledd blir feilen ved $f(1) < 0{,}0005$

Oppgave 2a)

$$\sum_{n=1}^{\infty} \frac{n}{(n-1)!} x^n$$, Bruker grense sammenlikning

$$\frac{\frac{(n+1)x^{n+1}}{(n-1+1)!}}{\frac{nx^n}{(n-1)!}} = \frac{(n+1)x}{n \cdot n} \xrightarrow{n \to \infty} 0 \text{ , For alle } x$$

$$\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = 0 \to \text{ konvergerer for alle } x$$

Oppgave 2b)

$$\sum_{n=1}^{\infty} \frac{n}{(n-1)!} x^n = \sum_{n=0}^{\infty} \frac{(n+1)}{n!} x^{n+1}$$

$$= \sum_{n=0}^{\infty} \frac{n x^{n+1}}{n!} + \sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$$

$$= \sum_{n=1}^{\infty} \frac{n x^{n+1}}{n!} + x e^{x}$$

$$= \sum_{n=1}^{\infty} \frac{x^{n+1}}{(n-1)!} + x e^{x}$$

$$= x^2 \sum_{n=1}^{\infty} \frac{x^{n-1}}{(n-1)!} + x e^{x}$$

$$= x^2 \sum_{n=0}^{\infty} \frac{x^{n}}{n!} + x e^{x}$$

$$\underline{\underline{= x^2 e^{x} + x e^{x}}}$$

Oppgave 3a)

$$T' = k(T-20) \qquad T(0) = 25$$

$$\frac{T'}{T-20} = k$$

$$\int \frac{T'}{T-20}\,dt = \int k\,dt$$

$$\ln|T-20| = kt + C_1$$

$$|T-20| = e^{kt+C_1} = e^{C_1} \cdot e^{kt}$$

$$T - 20 = Ce^{kt} \quad , \text{Hvor } C = \pm e^{C_1}$$

$$\underline{T = Ce^{kt} + 20}$$

$$T(0) = 25$$

$$25 = Ce^{k \cdot 0} + 20$$

$$\underline{C = 5}$$

$$\underline{\underline{T = 5e^{kt} + 20}}$$

3b) $T(3) = 22$

$$25 = Ce^{k\cdot 0} + 20$$

$$\underline{C = 5}$$

$$\underline{\underline{T = 5e^{kt} + 20}}$$

3b)

$$T(3) = 22$$

$$5e^{k\cdot 3} + 20 = 22$$

$$e^{3k} = \frac{2}{5}$$

$$3k = \ln\left(\frac{2}{5}\right)$$

$$\underline{k = \frac{\ln\left(\frac{2}{5}\right)}{3}}$$

$$T(t) = 5e^{\frac{\ln\left(\frac{2}{5}\right)}{3}t} + 20 = 21$$

$$\exp\left(\frac{\ln\left(\frac{2}{5}\right)}{3}t\right) = \frac{1}{5}$$

$$\frac{\ln\left(\frac{2}{5}\right)}{3}t = \ln\left(\frac{1}{5}\right)$$

$$\underline{\underline{t = \frac{3\ln\left(\frac{1}{5}\right)}{\ln\left(\frac{2}{5}\right)} = 5{,}27}}$$

Oppgave 4)

$$\int_1^3 x\ln(x)\,dx \approx S_4 \quad ; \quad 4 \text{ delintervaller}$$

$$h = \frac{b-a}{4} = \frac{1}{2}$$

$$\cancel{S_4 = h\left(\tfrac{1}{2}y_0 + y_1 + y_2 + y_3 + \tfrac{1}{2}y_4\right)}$$

$$S_4 = \frac{h}{3}\left(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4\right)$$

$$y_0 = 1 \cdot \ln(1) = 0$$

$$y_1 = \frac{3}{2} \cdot \ln\left(\frac{3}{2}\right) = 0{,}6082$$

$$y_2 = 2\ln(2) = 1{,}386$$

$$y_3 = \frac{5}{2}\ln\left(\frac{5}{2}\right) = 2{,}291$$

$$y_4 = 3\ln(3) = 3{,}296$$

$$S_4 = \frac{h}{3}(17{,}66)$$

$$\underline{S_4 = 2{,}943}$$

$y_4 = 3\ln(3) = 3{,}296$

$S_4 = \frac{h}{3}(17{,}66)$ ~~$\approx$~~

$$\underline{\underline{S_4 = 2{,}943}}$$

$$\left| \int_a^b f(x)\,dx - S_n \right| \leq \frac{K(b-a)^5}{180\, n^4}$$

Hvor K er definert slik at $|f^{(4)}(x)| \leq K$

for alle $x \in [a,b]$

$f(x) = x\ln(x) \rightarrow f'(x) = 1 + \ln(x)$

$f''(x) = \frac{1}{x}$ $\quad f^{(3)}(x) = -\frac{1}{x^2}$

$f^{(4)}(x) = \frac{2}{x^3}$

$K \geq \left| \frac{2}{x^3} \right|$ for alle $x \in [1,3]$

$K \geq \left| \frac{2}{1} \right| = 2$ $\quad f^{(4)}(x)$ er størst ved starten av intervallet.

$$\underline{K = 2}$$

$$\left|\int_a^b f(x)\,dx - S_n\right| \leq \frac{2(b-a)^5}{180\, n^4}$$

$$\left|\int_1^3 f(x)\,dx - S_n\right| \leq \frac{2^6}{180} \cdot \frac{1}{n^4} \leq 10^{-4}$$

$$n^4 \geq \frac{64}{180 \cdot 10^{-4}}$$

$$n \geq \sqrt[4]{\frac{64 \cdot 10^4}{180}}$$

$$n \geq 7{,}72$$

$n = 8$ gir feil mindre enn 10^{-4}